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THE EFFECTS OF AUDITOR REPUTATION IN MORAL HAZARD AND ADVERSE SELECTION SETTINGS

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THE EFFECTS OF AUDITOR REPUTATION IN MORAL HAZARD AND ADVERSE SELECTION SETTINGS

A DISSERTATION

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FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

BY

SRIKANT M. DATAR

JULY 1985

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

(Principal Advisor)

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Approved for the University Committee on Graduate Studies:

Dean of Graduate Studies & Research

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CHAPTER 1

INTRODUCTION

This dissertation analyzes two models of auditing. The first is a multi-period, game-theoretic model of auditing with incomplete information. The role of the audit is to produce information upon which contracts between owners and managers are based. We model the auditor as a rational economic agent taking actions under moral hazard. We derive endogenously conditions for reputation formation by auditors that mitigate the moral hazard problem. The second model is a single period model of adverse selection. An entrepreneur wishes to raise capital in the capital market. The role of the audit is to produce information that will assist potential investors in valuing the firm. In the next two sections we discuss these models in greater detail. Our plan of analysis is presented in Section 1.3.

1.1 THE MULTI-PERIOD GAME THEORETIC MODEL

Agency theory analyzes situations in which the owner of a production process contracts with a manager to take productive acts on the owner's behalf. It is typically assumed that the manager's effort is unobservable and that the support of the outcome distribution is independent of the agent's effort. Contracting is necessarily confined to the jointly observed outcome. In our model, in addition to the productive act being unobservable, the actual outcome is also unobserved by the owner. Contracting must therefore be confined to the outcome reported by the manager. This raises the following questions. Is there a demand for additional information that will confirm the manager's reports? If so, how is this information generated?

The demand for confirmation of financial reports arises because otherwise management has incentives to misrepresent the financial condition of the firm. The incentives for management to do so arise because owners use financial reports to evaluate management's performance. Performance evaluation is desirable because management takes actions under moral hazard.

Auditing may be demanded if the audit report can be used to 'help' motivate truthful reporting. The owner hires an auditor to produce information used in contracting with the manager. The audit report affects payments to the manager. The structure of these payments is determined endogenously and affects both the manager's productive act and report. A key feature of our model is that auditors are modeled as rational economic agents with preferences and beliefs. We assume that auditors take acts and issue reports under conditions of moral hazard, with their behavior represented by expected utility maximization.

Our objective is to model explicitly and endogenously the auditor's incentives to develop a reputation that mitigates the auditor's moral hazard problem. Reputation formation is essentially a multi-period construct and auditing is a multi-period activity. Our model of auditing is a multi-period game-theoretic model with incomplete information. As we shall see in Chapter 2, this distinguishes our model from all others in the accounting literature.

1.2 THE ADVERSE SELECTION MODEL

In most economic models of auditing analyzed in the accounting literature, the demand for auditing is modeled to reduce the moral hazard problem in a principal-agent relationship. In our second model, we consider an adverse selection setting. An entrepreneur wishes to raise capital in the capital markets. The entrepreneur has superior information about project quality but investors cannot distinguish between various projects. As argued by Akerlof (1970), where the supply of poor projects is large relative to the supply of good projects, venture capital markets may fail to exist.

For projects of good quality to be financed information transfer must occur. Leland and Pyle (1977) show that the fraction of equity in the firm, ψ, retained by the entrepreneur, can serve as a noiseless signal of the quality of the project, μ. Signaling μ to the market via ψ results in a welfare loss to the entrepreneur relative to the case where μ can be costlessly communicated. The loss in welfare creates an opportunity for the auditor to collect a fee to provide a more efficient mechanism for verification of project quality.

Our objective is to examine conditions under which hiring an auditor is more efficient for the entrepreneur than signaling μ via ψ alone. After observing the audit report $\hat{\mu}$, investors assess a posterior probability distribution for μ given $\hat{\mu}$. Higher audit quality corresponds to a 'tighter' posterior probability distribution of μ. Our model suggests conditions under which hiring an auditor is beneficial in an adverse selection context and provides some insight into the existence of a few large public accounting firms. As we shall see in Chapter 2, the adverse selection nature of our model within a market setting distinguishes our model from most others.

1.3 ORGANIZATION OF DISSERTATION

The dissertation is organized as follows. In Chapter 2, we review the literature on multi-period principal agent models, models of reputations, models of adverse selection in markets and some studies that use analytical models to address auditing questions. Chapter 3 presents a multi-period game theoretic model of auditor reputation. The model is a repeated principal agent model where only the manager observes the outcome and makes a report about this information to the principal. The owner hires a second agent, the auditor, to report on the manager's actions and information. Like the manager, the auditor is modeled as a rational economic agent. We therefore have a two-agent, multi-period agency model. As in Antle (1980, 1982) this gives rise to the problem of subgame dominance in the sense that the agents may not find it in their best interest to take the actions that the owner wants them to take. We argue that auditor reputations may alleviate this problem.

Chapter 4 presents the adverse selection in markets model. We consider fully separating or informationally consistent equilibria. In the absence of auditing, the entrepreneur can communicate project quality μ, via ψ, the fraction of equity in the firm retained by the entrepreneur. The entrepreneur hires the auditor to provide a report $\hat{\mu}$ on μ. We examine the entrepreneur's optimal choice of auditor and conditions under which it is more efficient for the entrepreneur to hire the auditor rather than signal μ via ψ alone. Chapter 5 summarizes the dissertation and suggests directions for future research.

CHAPTER 2

LITERATURE REVIEW

The purpose of this chapter is to review the literature that is relevant to this dissertation. This chapter is organized around the four main streams of research that are most pertinent: multi-period agency theory, the theory of reputations, signaling theory and economic models of auditing. We focus only on those auditing studies that use formal economic models to explore observed auditing phenomena (such as models that attempt to explain due care standards, the concern for reputation, legal liability and the structure of the audit industry).

This Chapter is divided into four sections. We start by reviewing the auditing literature. In Section 1 we review papers that deal with the demand and supply of audit services. Section 2 is a review of multi-period agency theory. In Section 3 we review papers from the reputations literature that are particularly relevant to this dissertation. Section 4 is a review of adverse selection and signaling models.

2.1 ECONOMIC MODELS OF AUDITING

Most economic studies of auditing have been single period models. In many of these studies the auditor is modeled as a non-rational mechanistic technology rather than as a utility maximizing agent. Antle (1980) models the auditor as a rational economic agent and highlights the fact that a three person game has a non-trivial two-person subgame between the manager and auditor. In Chapter 3 we extend Antle's model to multiple periods and show that reputation effects can resolve some of the difficulties raised by Antle. Since Antle (1980, 1982) is directly pertinent to this dissertation we shall review this work in greater detail than we will the other studies.

We first review various papers that focus on the demand for auditing: Evans (1980), Fellingham and Newman (1979), Ng (1978) and Ng and Stoeckenius (1979). These authors see a demand for auditing arising from the information asymmetry and moral hazard in the owner-manager relationship. The crux of the moral hazard problem in our model and in many of these papers is the existence of nonpecuniary returns to the manager for supplying productive inputs. We next review the work of Hamilton (1977). The demand for auditing in Hamilton's model arises due to adverse selection problems. We briefly review Scott (1975) and Magee (1977), who focus on the auditors' loss functions, and DeAngelo (1981a,b) who examines a multi-period audit setting. Finally, we review Antle (1980, 1982).

Fellingham and Newman (1979) model a manager privately choosing a level of investment for the firm and consuming the difference between the amount of capital raised from owners and the amount invested. They do not assume nonpecuniary returns to the manager. If no monitoring is available Fellingham and Newman show that the input level chosen will not maximize profits. Auditing is a means for the enforcement of contracts between owners and managers. If monitoring is available and sufficiently economical, the manager will choose to be monitored and guarantee the input level.

Ng (1978) argues that demand for auditing results from a conflict of interest between the manager and owner of a firm over the information (accounting) system employed to report the results of operations to the owner. Ng shows that if the manager were free to select any reporting method he likes, he would select one which is as coarse and as positively biased as possible whereas the owner would prefer a reporting system that has less bias and is finer. Consequently, a role of Generally Accepted Accounting Principles (GAAP) is to limit the set of acceptable reporting functions from which the manager may choose. Ng concludes that a major function of auditing is to provide an expert opinion as to whether the firm's reporting policy is consistent with GAAP. Auditing serves as a detection mechanism. Ng does not allow for any disutility for acts and restricts attention to contracts that pay the manager a proportion of the reported amount. There is no analysis of the welfare effects or the optimality of such contracts.

Evans (1980) studies costly perfect auditing in a model of uncertain production and nonpecuniary returns. He models a situation where the principal cannot observe the outcome or the managerial inputs. The agent can, after observing the outcome, hire an auditor who for a fixed fee will report the payoff and effort perfectly. Evans shows that contracts which call for investigation regions that agree with the regions in which investigation is ex-post optimal for the manager weakly Pareto-dominate all other contracts. He establishes that the agent will have himself audited when the payoff is in the lower end of the outcome interval and that the optimal contract is dichotomous in the manager's action over the verification region. Evans' model is one of costly conditional auditing which ours is not. However, the auditor is not modeled as a rational economic agent. Second, Evans assumes rather than proves the form of the optimal contract.

Ng and Stoeckenius (1979) model a situation in which neither the firm's payoff, the manager's effort nor the realization of the random state of nature can be observed by the owner. The outcome is observed by the manager who reports some signal (such as reported earnings versus actual earnings) to the principal. Similarly, the auditor gathers evidence and first generates an opinion for himself and secondly a report based on the evidence for the principal. Ng and Stoeckenius show that, without auditing, the only equilibria in which reporting is truthful are equilibria with pure wage contracts. An auditing technology is modeled as a probability of detecting untruthful reporting. This probability increases with the size of the error and the intensity of the audit chosen by the principal. If any discrepancy is discovered the manager suffers (a possibly infinite) penalty. Under these assumptions they show that costless auditing will motivate the manager to exert reasonable effort and report the payoff truthfully while preserving Pareto optimal risk sharing in a second best sense. Once again the audit is done without any incentive problem.

Whereas the demand for auditing in the papers we have reviewed so far arises due to moral hazard in the owner manager relationship, Hamilton (1977) considers a model in which the demand for auditing arises because the agent has better information about the state of nature than the principal (the adverse selection problem). He models a situation in which firm managers have superior information about the firm's securities. Firm managers who know the true state of their firm's securities is poor will tend to bias their reports. Realizing this the market will undervalue the firm. This creates conditions under which firm managers may wish to hire an auditor to reveal information about the firm's securities in order to convince owners the reports are unbiased and to ensure the market does not undervalue the firm.

Hamilton does not consider the optimality of value maximizing behavior by the manager. The auditor's incentives are also not considered. We model a situation similar to Hamilton's in Chapter 4 and consider the optimality of value maximizing behavior. We do not explicitly model the auditor's incentives not to shirk. In a multi-period model, arguments similar to those used in Chapter 3 could be used to motivate non-shirking by the auditor. We also restrict ourselves to linear (equity) contracts.

Scott (1975) analyzes the role of the auditor as a sharer of risk with the users of financial statements. He models investors as making consumption-investment decisions under uncertainty. The user is faced with an uncertain distribution on returns from the firm. The auditor provides the investor with a point estimate of the firm's return. Scott equates the auditor's loss function with the losses suffered by a typical financial statement user who relies on the financial statements for an investment decision. To the extent the financial statements contain errors the (representative) investor's decision will be non-optimal and the user will suffer a loss. Scott derives the amount of the user's loss as a function of the extent of error and takes the professional auditor's loss function as the expectation of the user's loss function. This means that there is no differential information between the auditor and investors and therefore no real reporting problems. Consequently, the role of the auditor is unclear. Moreover, there is no fieldwork or investigative work in Scott's model and auditors are not modeled as rational economic agents acting under moral hazard.

Magee (1977) models the auditor as a rational economic agent. He sees the auditor as mitigating the moral hazard problem resulting from the owner's inability to monitor the firm's outcome and the manager's actions. The auditor's role is to detect any misreporting by the manager. Magee points out that the amount of auditing an auditor does creates nonpecuniary costs on the auditor, an aspect missing from Scott's model, and thereby creates moral hazard on the auditor's part. However, he replaces the manager with a probability distribution and does not model the manager as a strategic player. Consequently, he does not consider the effect of auditing on managerial behavior. Also, there is no reporting by the auditor in Magee's model.

DeAngelo (1981a) examines a multi-period model of auditing. She considers the impact on fees charged by auditors because of technological start-up costs (for the initial audit) and transactions costs of switching auditors. Such costs result in auditors earning quasi-rents in future periods. Under the competitive markets assumption audit firms earn zero profits over the multi-period horizon. Consequently, DeAngelo argues that 'low balling' (that is charging lower audit fees) in the initial period is a competitive response to the expectation of future quasi-rents to incumbent auditors. That is, 'low balling' in the initial period is the process by which auditors compete for these advantages.

DeAngelo (1981b) defines the quality of audit services as the market-assessed joint probability that a given auditor will both (a) discover a breach in the client's accounting system, and (b) report the breach. She defines auditor independence as the conditional probability of reporting a discovered breach. DeAngelo argues that the existence of client-specific quasi-rents to incumbent auditors lowers the optimal amount of auditor independence because the auditor does not want to risk losing the future stream of quasi-rents by displeasing the client. However, she suggests that auditors with a greater number of current clients have reduced incentives to 'cheat' in order to retain any one client. If the auditor 'cheats' and is caught he stands to lose some portion of the present value of the quasi-rents specific to other current clients of the auditor. This leads DeAngelo to conclude that rational investors perceive larger firms as signaling a higher quality of audit.

DeAngelo (1981a,b) does not explicitly model the various players in the game as strategic players. She does not specify the incentives and strategies of the various players and therefore her arguments are not equilibrium arguments. Likewise, DeAngelo assumes rather than proves that larger audit firms have greater incentives to maintain their independence. The strategic responses and threats of the various players in the game are not considered.

Antle (1980,1982) analyzes a single period game-theoretic model where the manager, owner and auditor are all strategic players. The manager and auditor each take actions under moral hazard. They also issue reports to the principal. In addition to motivating the manager, the owner must now design a contract to motivate the auditor to exert reasonable diligence. Antle's formulation allows the owner to take legal action against the auditor (lack of "due care").

Antle explicitly recognizes that, unlike the two-person principal agent model which results in a trivial subgame (since the principal moves first), the three-person principal-agent-auditor game is non-trivial because each move of the owner determines a two-person game to be played by the manager and auditor. Of all the Nash equilibria in this two-person game that satisfy the minimum utilities of the manager and auditor, the owner's maximization problem entails choosing the one that maximizes the owner's expected utility. But in general all Nash equilibria of a two-person game do not provide the players with the same expected utility. The subgame equilibrium that the principal prefers may not be the one that maximizes the manager's and auditor's expected utilities. Antle shows an example in which the Nash equilibrium preferred by the owner in the auditor-manager subgame is Pareto dominated (in that subgame) by alternative Nash strategies of the auditor and manager that make the owner worse off.

Antle suggests that the owner's maximization problem should be altered by adding some "collective rationality" constraints to at least guarantee the agents do not play a dominated subgame equilibrium. However, since a tractable formulation of the constraints seems difficult, Antle restricts his analysis to pure strategy (and possibly dominated) equilibria.

Antle uses his model to investigate auditors' legal liability, due care standards and notions of auditor independence. He shows that a necessary condition for litigation to be useful is that the court's report be informative about the auditor's act. The optimal litigation policy calls for suing always or never, conditional on the auditor's and manager's reports. He defines due care contracts as contracts that penalize the auditor for negligence and shows that they are weakly Pareto-dominant.

Antle formulates three definitions of auditor independence suggested by the literature and analyzes their implications. Independence (1) states that optimal auditor payments do not depend on the audit report. A sufficient condition for this is eliminating moral hazard on the auditor's investigative act. Independence (2) states that there is an optimal auditor's contract which is truth-inducing. This follows from Antle's truth-inducing representation of the owner's problem. Independence (3) states that the auditor-manager subgame is played noncooperatively which is the solution concept assumed in the analysis.

In Chapter 3 we model the mechanism by which information is produced and allow for moral hazard on the auditor's and manager's action choices and reporting decisions. We analyze a multi-period game theoretic model of the audit relationship where the owner, manager and auditor are all strategic players and explicitly model auditor reputation for hard work and independence. Using a simple model, we employ the notion of sequential equilibrium (which we discuss in Section 2.3) to show that the auditor and manager will not play a dominated subgame equilibrium. We then examine the question of auditor independence and the auditor's incentives to maintain a reputation. The model contributes to the auditing literature by explicitly considering: (i) the owner, manager and auditor as strategic economic agents in a multi-period game; and (ii) the role of auditor reputation.

In Chapter 4, we examine a single-period adverse selection setting in which an entrepreneur with superior information wishes to sell shares to outside investors. Our model is one of the first models in the auditing literature that examines the benefit from hiring an auditor to reduce adverse selection problems in a market setting. We analyze optimal economic arrangements among agents with and without auditing.

2.2 TWO-PERSON MULTI-PERIOD AGENCY MODELS

In this section we review the two-person multi-period agency models directly relevant to this dissertation. We review the work of Lambert (1983) and Rogerson (1985) in greater detail and briefly review the work of Fellingham, Newman and Suh (1983), Holmström and Milgrom (1984), Radner (1981a,b,c), Rubinstein and Yaari(1983) and Holmström (1982). Throughout this section we assume the principal cannot observe the agent's actions, so that the optimal contract can only be based on the observed cash flow. Our main area of interest in this section is to understand the nature of optimal contracts in multi-period agency models.

The two-person multi-period agency model considers a situation where one person (the principal) wishes to hire another person (the agent) to work for him over several periods, 1...,T. The principal contracts with the manager to exert effort $a_t \epsilon A_t$ in period t which together with a random uncontrollable state $\bar{\theta}_t$ results in an outcome (cash flow) $x_t = x_t(a_t, \bar{\theta}_t)$. The agent's compensation in period t can depend only on variables which are jointly observable at the end of period t. The compensation function s_t in period t is a function of the agent's performance in periods one through t. The strategy for the principal is a sequence of functions, $s = [s_1(x_1), \; s_2(x_1,x_2) \; \ldots \; s_T(x_1,\ldots,x_T)]$. The agent's strategy is to decide how much effort $a_t \epsilon A_t$ to supply in each period. If the agent precommits to remaining with the firm for all T periods, the agent's choice of effort can depend on his performance in the prior periods. A strategy for the agent can therefore be represented as $\underline{a} = [a_1, a_2(x_1), \; \ldots, \; a_T(x_1,x_2,\ldots,x_T)]$.

The principal and agent are assumed to possess utility functions which are additively separable by periods. The principal's utility function is denoted by $G = \sum_{t=1}^{T} G_t[x_t - s_t]$, and the agent's by $U = \sum_{t=1}^{T} U_t = \sum_{t=1}^{T} [W_t(s_t) - V_t(a_t)]$. It is further assumed that $G_t'(\cdot) > 0$, $G_t''(\cdot) \le 0$, $W_t'(\cdot) > 0$, $W_t''(\cdot) < 0$, $V_t'(a_t) > 0$ and $V_t''(a_t) > 0$.

If the sequence of events in the multi-period agency problem is that the principal selects a first period contract, the agent responds with first period effort, the principal selects a second period contract, the agent chooses a second period effort etc., Lambert (1981) shows that the T period game is equivalent to T separate one period games. That is, in each period the principal and the agent act just as they would in a single period game. The sharing rule and the effort selected in one period do not depend on either the history of the game or anything that will occur in future. The separability of the utility functions and the production functions over time and the fact that the outcome in one period has no real effect on the utility function, production function or the minimum utility level in a subsequent period drives this result. Lambert (1981, 1983) models a game in which the principal announces and precommits to the entire sequence of contracts for the T periods at the beginning of the first period. The agent then responds by selecting a sequence of actions as the game unfolds. The following time line illustrates the sequence of choices in the model when T = 2:

Principal chooses	Agent chooses	Both observe	Agent chooses	Both observe
$s_1(x_1), s_2(x_1, x_2)$	a_1	x_1	$a_2(x_1)$	x_2

The principal's problem in the two period model can be defined as follows:

$$\text{(2.1)} \qquad \underset{s_1(x_1),\, s_2(x_1,x_2),\, a_1,\, a_2(x_1)}{\text{Max}} \; E[G_1[x_1 - s_1(x_1)] + G_2[x_2 - s_2(x_1, x_2)]]$$

subject to

(2.2) $E[U_1[s_1(x_1),a_1] + U_2[s_2(x_1,x_2),a_2(x_1)]] \geq 0$

(2.3) $[a_1,a_2(x_1)] \in \text{argmax } E[U_1[s_1(x_1),a_1] + U_2[s_2(x_1,x_2),a_2(x_1)]]$

Using a dynamic programming approach and replacing the argmax condition by the first-order conditions, Lambert rewrites constraint (2.3) as follows:

(2.4) $\sum U_2[s_2(x_1,x_2)]p_2'(x_2|a_2(x_1)) - V_2'[a_2(x_1)] = 0$ for each x_1

(2.5) $\sum [U_1(s_1(x_1)) + EU_2(s_2(x_1,x_2),a_2(x_1))]p_1'(x_1|a_1) - V_1'(a_1) = 0$

where $p_t'(x_t|a_t)$ is the derivative of the probability density with respect to a_t.

Solving the above program, Lambert shows that the optimal contract bases compensation in one period, in part, on performance in earlier periods. Even if the agent cannot precommit to a multi-period contract, it is optimal to supplement current period incentives with future period incentives, though not as extensively as in the case in which the agent is also able to precommit to remaining with the organization.

Rogerson (1985) assumes both the principal and the agent can commit to a long term contract and like Lambert (1983) shows that in a Pareto-efficient contract, whenever an outcome has any effect on the current wage it must also have an effect on future period wages. The explanation of the role of time in these models lies in the dual role of the wage contract; providing incentives for the agent to choose the desired action and allocating risk away from the agent to the less risk-averse principal. By precommitting to a long term contract, the principal can choose to evaluate the agent's performance over the entire history of his employment. Consequently, some of the "noise" which obscures the agent's actions may wash out over time. The Pareto-efficient contract always spreads incentive payments into the future so that, in some sense, more incentive maintenance is occurring in the later periods of the relationship. Rogerson (1985) shows that the expected wage payment will rise (fall) over time if the agent's inverse marginal utility is concave (convex).

Fellingham, Newman and Suh (1985) show that Lambert's result is not completely robust to the nature of the agent's utility function. They identify necessary and sufficient conditions that lead to memoryless contracts. They define memoryless contracts as contracts in which the agent's compensation in any period does not depend on his performance in past periods. In such cases a single period analysis is all that is required. They show that the optimal contract will have no memory if the agent's utility function is multiplicatively separable and exhibits intertemporal constant absolute risk aversion. In such a case the agent is only interested in his total compensation rather than the risks associated with his compensation in each period.

Holmström and Milgrom (1984) consider a repeated principal-agent model in which the agent chooses a distribution P over an outcome set with a finite number N+1 of distinct outcomes incurring a cost, C(P). The agent has a constant coefficient of absolute risk aversion. The principal's objective is to maximize expected terminal wealth. After each period, the agent observes the actual outcome before the next period begins. A strategy for the agent is a discrete time stochastic process that specifies the action to take at each period as a function of the history up to that time. Holmström and Milgrom show that the optimal dynamic incentive scheme is equivalent to instituting the optimal single period scheme in each period separately. They further show that the optimal sharing rules are ones that pay the manager a linear function of the outcomes at the end of each period.

Radner(1981c) and Rubinstein and Yaari (1983) consider an infinite period game without discounting. They show that a first-best solution is attainable since the principal can eventually detect any systematic shirking by the agent by comparing the agent's average output with that which would be expected if the agent had chosen the agreed upon effort each period. They analyze a simple dichotomous contract in which the agent is offered the first-best sharing rule in period t if the agent's average performance up to period t-1 is "within bounds" and a penalty contract if his performance is "outside the bounds".

Radner (1981b), uses the concept of epsilon equilibrium to show that for any Pareto-efficient outcome of the one-period game that dominates a one-period Nash equilibrium, there exists a time horizon sufficiently long such that the non-cooperative epsilon equilibrium of the repeated game yields the principal and agent an average per period expected utility that is arbitrarily close to the expected utility of the first best one-period solution. Radner (1981a) shows that a first-best solution is nearly achievable if the discount rate for future payoffs is close to 1.

Holmström (1982) studies a model in which a manager's effect on output in each period is the sum of three effects: a fixed component or skill, a component chosen by the manager called effort, and a random noise term. The owner can not distinguish between skill and effort from his (noisy) observations of output. The skill component is assumed to move over time as a random walk. The manager's wage is equal to his expected contribution to output from skill and effort. The manager must provide sufficient effort to prevent the owner from inferring that his skill contribution is low. For a class of models, Holmström shows that a stationary equilibrium induces a supply of effort that is close to the efficient one to the extent that the manager's interest rate is close to zero, and the variance of the skill term's random walk is large relative to the variance of the noise term.

The multi-period agency literature is relevant to the auditing problem that we model in Chapter 3. We consider a multi-period model in which both the manager and auditor take actions and issue reports under moral hazard. We use the techniques employed in agency theory to determine optimal multi-period contracts for the manager and auditor.

Our model differs from the multi-period agency literature in three ways. First, the actual cash flow in our model is not observed by the owner. It is reported to the owner by the manager. This introduces the additional complexity of moral hazard on the manager's report. Second, our model contains two agents, the manager and auditor, who take actions under moral hazard and interact non-trivially over multiple periods. This results in the problem of subgame dominance that we referred to in Section 2.1. Third, our model is one of incomplete information about the auditor. This results in incentives to develop a reputation. We discuss this literature in the next section.

2.3 REPUTATIONS

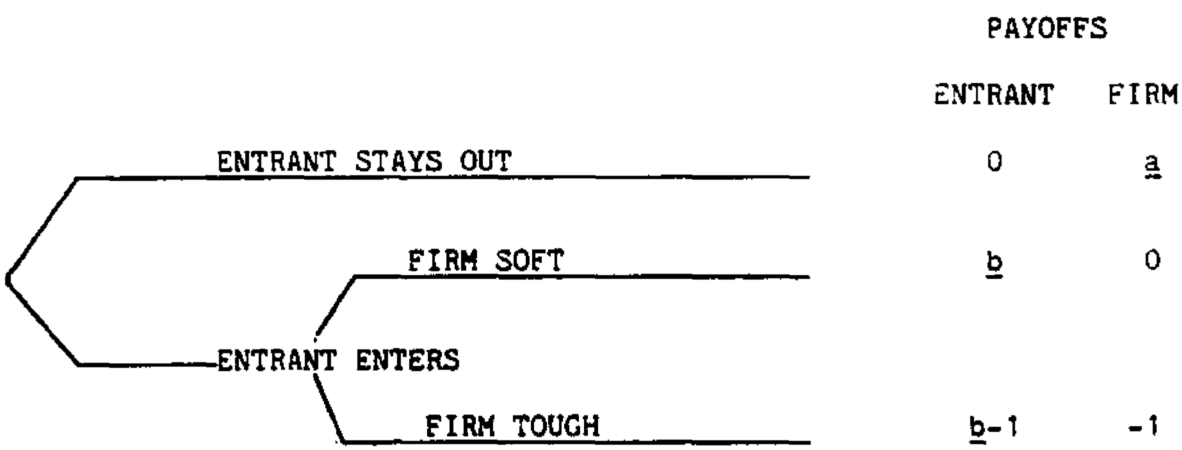
Models of reputations abound in the extant literature (see Wilson (1983b) for an excellent survey). We do not propose to review many of these models here. Instead we briefly comment on Holmström (1982) and then focus on Kreps and Wilson (1982b), a paper that is especially relevant to this dissertation.

Holmström (1982), which we reviewed in the previous section, illustrates a general feature about the role of reputations in providing incentives in labor markets. To the extent that skill and effort are substitutes and imperfectly observable, long-term contracts with the agent's remuneration contingent on the history of measured performance provide incentives for workers to invest effort in building a reputation for quality (skill).

The equilibrium notion employed by Kreps and Wilson (1982a) is that of sequential equilibrium (Kreps and Wilson, 1982b). Since sequential equilibrium is the solution concept employed throughout this dissertation we briefly review it here. Sequential equilibrium invokes the criterion of sequential rationality: the strategy of each player starting from each information set must be optimal starting from there according to some assessment over the nodes in the information set and the strategies of everyone else. Part of this computation is the construction in each circumstance of a probability assessment for those events about which the player is uncertain. One of the implications of optimality is that this probability assessment is a conditional probability satisfying Bayes' rule whenever the conditioning event (the circumstance in which the player finds himself) has non-zero probability according to the supposed strategies of the other players.

Kreps and Wilson show that perfect equilibria are generically the same as sequential equilibria for a broad class of games. A particular strategy combination is a perfect equilibrium if there exists a sequence of positive combinations of strategies converging to it such that each player's designated strategy is an optimal response to each member of the sequence. The motivation is that each player's strategy be a robust best response in the sense that it is an optimal response even for some small probability that others may err.

We are now in a position to review Kreps and Wilson (1982a). Kreps and Wilson consider reputation effects in a game in which a monopoly firm plays in sequence against n opponents (entrants) the following stage game.



$0 < \underline{b} < 1$, $\underline{a} > 1$

Each entrant maximizes its expected payoff; the firm maximizes the sum of its payoffs over the n stage games. The only sequential (perfect) equilibrium has the entrant entering and the firm being soft at each stage of the n stage game.

Kreps and Wilson argue that if the entrants initially assign a probability $\underline{\delta} > 0$ that playing tough is the firm's only feasible action, that is, the firm is 'strong' rather that 'weak', the sequential equilibrium is substantially altered. They define $\bar{q}_t$ to be the entrants' probability assessment that the firm is strong when t stages remain, i.e., $\bar{q}_t = \underline{\delta}$. Then the weak firm plays tough if $\bar{q}_t \geq \underline{b}^{t-1}$ and otherwise it plays tough with a probability determined by the condition that $\bar{q}_{t-1}$ will be $\underline{b}^{t-1}$ if it plays tough and zero if it plays soft. When $\bar{q}_t \geq \underline{b}^{t-1}$, both the strong and weak firms play tough so that no information is gained about the type of firm and $\bar{q}_{t-1} = \bar{q}_t$. The optimal strategy for the entrant is to stay out if $\bar{q}_t > \underline{b}^t$, to enter if $\bar{q}_t < \underline{b}^t$ and to randomize by staying out with probability $1/\underline{a}$ if $\bar{q}_t = \underline{b}^t$.

Kreps and Wilson interpret $\bar{q}_t$ as the firm's reputation. They use the equilibrium to show how reputation considerations affect the players' choice of actions. Entrants are playing optimally, since if $\bar{q}_t \geq \underline{b}^{t-1}$ entrant t expects the firm to be tough and so stays out. If $\bar{q}_t \in (\underline{b}^t, \underline{b}^{t-1})$, the firm will be soft with positive probability, but with probability less that $1-\underline{b}$, so again it is better for the entrant to stay out. If $\bar{q}_t = \underline{b}^t$, the firm will be soft with probability $1-\underline{b}$, and the entrant is indifferent. If $\bar{q}_t < \underline{b}^t$, the probability of the firm being soft exceeds $1-\underline{b}$ and the entrant enters.

Kreps and Wilson then show that the strategy of the weak firm is optimal. The expected payoff to the firm when t stages remain, assuming the above strategies, is 0, 1, or $\underline{a}(t-\tau(\bar{q}_t)+1)$ according as $\bar{q}_t$ is less than, equal to, or greater than $\underline{b}^t$, where $\tau(\bar{q}_t) = \inf [\tau | \bar{q}_t > \underline{b}^t]$. If entry occurs at stage t, by playing soft, the firm receives zero both in this stage and in the rest of the game (since $\bar{q}_{t-1}$ will be set equal to zero). By playing tough the firm receives -1 in this stage. If the firm is tough and if $\bar{q}_t \leq \underline{b}^{t-1}$ the expected payoff over all subsequent stages of the game exactly equals 1. If the firm is tough and if $\bar{q}_t > \underline{b}^{t-1}$, the future expected payoffs are greater than 1. Therefore, playing tough is optimal for the weak firm. The above analysis amounts to a verification that the firm's strategy solves the associated dynamic programming problem, given the way in which reputation evolves according to Bayes' rule.

The key idea is that if the weak firm's reputation for possibly being strong is sufficiently large compared to the number of stages to be played, it pays the weak firm to mimic the strong firm so as to deter entrants from entering and being roughed up for demonstration purposes. The firm's reputation is an asset that it acts to maintain, even if doing so is costly in the short term, so as to reap the benefits in subsequent stages of the game. Indeed, as the number of stages in the game goes to infinity the weak firm obtains an expected payoff per stage of $\underline{a}$, whereas if it revealed itself its payoff would be 0 forever after.

Milgrom and Roberts (1982b) analyze a similar situation. They emphasize that reputation effects can arise whenever any of the players has information that is not common knowledge among all of them. They consider a situation in which each entrant may know the firm is weak but may not be certain whether his fellow entrants have this information. The firm will then play tough in order to maintain its reputation among the other entrants. Consequently, each entrant's best response is not to enter.

Kreps, Milgrom, Roberts and Wilson (1982) consider a finitely repeated version of the symmetric Prisoners' Dilemma. They show that if Player A entertains a positive probability that player B will cooperate unless A does not, the equilibrium in the game calls for both players to cooperate over a large number of periods in the game. Similar reputation effects are studied by Moorthy (1980) in the context of a durable-good monopoly. Maskin and Fudenberg (1983) develop a general theory of finitely repeated games with incomplete information. The studies cited earlier can be viewed as special cases of the general theory.

It is important to stress the equilibrium notion that underlies the above analysis. The critical requirement is that of sequential rationality: At every stage of the game the player's subsequent strategy must be optimal with respect to some assessment of the probabilities of all uncertain events, including any preceding but unobserved choices of the other players. Essentially, the notion of equilibrium is broadened from simply a <u>strategy</u> to two types of probability assessments by the players: the <u>beliefs</u> of a player concerning where in the game tree he is when it is his turn to move; and his conjecture concerning what will happen in the future, given by his strategy. Specifying beliefs off the equilibrium path allows us to verify that the player's own strategy is optimal at each stage of the game.

Our model of auditor reputations in Chapter 3 is similar in spirit to that of Kreps and Wilson (1982b). However, we consider a three person game and attempt to develop a concept of reputation in labor markets characterized by moral hazard. Reputation effects are developed within an agency setting. This considerably complicates the analysis because reputation formation depends not only on the strategies of the various players but also on the random realizations of states of nature. Our game is a dynamic game rather than a repeated game. We explicitly analyze how various players in the agency benefit from reputation formation. The key insight here is that a player other than the player building a reputation benefits from reputation formation. Finally, we extend the Kreps and Wilson analysis to a situation where the strong firm does not always play tough. We obtain some qualitatively different results about reputation formation.

SECTION 2.4 ADVERSE SELECTION AND SIGNALING

Numerous markets are characterized by informational differences between buyers and sellers. In financial markets information asymmetries are particularly pronounced. Entrepreneurs possess superior inside information about the quality of the projects for which they seek financing. Investors only have some distribution over the quality of projects. Akerlof (1970) was the first to point out that where substantial information asymmetries exist and where the supply of poor quality projects is large relative to the supply of good projects, venture capital markets may fail to exist.

Spence (1973) shows that it is possible for entrepreneurs to signal the value of their projects. The critical assumption is that the cost of the signal is negatively correlated with project quality. Leland and Pyle (1977) demonstrate that the signal ψ, the fraction of equity in the project retained by the entrepreneur satisfies Spence's condition and can be used to signal the project's mean, μ. Since our analysis extends that of Leland and Pyle, we review the Leland and Pyle paper in considerable detail in Chapter 4.

Hughes (1985) analyzes a Leland and Pyle type model in which the mean and variance of the project are both unknown to investors so that μ can not be perfectly signaled via ψ alone. Hughes considers two signals: The fraction of ownership in the firm, ψ and direct, though not necessarily full, disclosure about μ. Direct disclosure is a credible means of communication in Hughes' model because penalties are assumed for false disclosure which are of sufficient size to induce truthful disclosure. The two signals, disclosure and firm ownership, unambiguously communicate firm value. Like Hughes, we consider two signals. However, the variance of the project is known in our model. Our objective is to examine conditions under which a second imperfect signal improves efficiency.

Two types of signaling equilibria are discussed in the literature. See for instance Spence [1973], Saloner [1982] and Cramton [1982].

(i) A fully separating or informationally consistent equilibrium. In this equilibrium, the quality of the project is correctly identified by the market via the signal ψ for all projects undertaken by the entrepreneur.

(ii) A pooling or partially pooling (or partition) equilibrium. In this equilibrium, the quality of the project assessed by the market with a given signal ψ_0 is equal to the average quality of those projects that are signaled via ψ_0. In other words, the signal does not provide perfect information about the quality of the project. We sometimes refer to such equilibria as weakly informationally consistent. A weakly informationally consistent equilibrium may separate out some subsets of projects while other projects are in one or more heterogeneous pools.

In the case of a continuum of types, Riley (1979) examines the viability of both informationally consistent and weakly informationally consistent equilibria. Riley defines a Nash equilibrium as follows: A set of (weakly) informationally consistent contracts is a Nash equilibrium if there exists no alternative price offer for shares which, if made by a single investor, would result in his purchasing shares of firms with expected market value in excess of the price paid. Riley proceeds to show that there is no weakly informationally consistent Nash equilibrium. Whenever projects of heterogeneous quality are pooled, there exist alternative profitable price offers which draw away the higher quality projects. In general there is no informationally consistent Nash equilibrium either. In the special case (such as ours) where there is a mass point at the lower end of the distribution and the cumulative distribution function of projects is strictly concave, the informationally consistent equilibrium is a Nash equilibrium.

Riley proceeds to define a reactive equilibrium as follows: A set of price offers by investors is a reactive equilibrium if for any additional offer by an investor which generates an expected gain to the investor making the offer, there is another offer which yields a gain to the second investor and losses to the first. Moreover, no further addition to the set of offers generates losses to the second investor. Riley argues that if investors learn to anticipate such reactions, no investor will make any alternative offer and the unique reactive equilibrium will be the Pareto-dominating separating equilibrium. Kreps (1984) argues in favor of the separating equilibrium as being a stable equilibrium (see Kohlberg and Mertens (1982)), which the pooling (and partially pooling) equilibria are not.[1]

In Chapter 4 we present an adverse selection model. An entrepreneur possesses superior inside information about the quality of the project for which he seeks financing whereas investors only have some distribution over the quality of the project. The entrepreneur employs an auditor to communicate information about the project's quality to investors. The model contributes to the auditing literature by explicitly considering conditions under which hiring an auditor increases the efficiency with which private information can be communicated.

The adverse selection and signaling literature is helpful in many ways. First, we obtain conditions under which there will be no demand for auditing. Second, we employ the rational expectations approach commonly used in solving adverse selection problems.

Our model differs from the extant adverse selection literature in two ways. First, we introduce the auditor to produce additional information in the entrepreneur-investors game and derive a sequential equilibrium in the modified game. Second, we analyze the optimal choice of auditor by the entrepreneur.

ENDNOTES TO CHAPTER 2

1 The general idea of perfect equilibrium (Selten, 1975) and sequential equilibrium (Kreps and Wilson, 1982b) is that a Nash equilibrium set of strategies should be stable in the sense that there is <u>some</u> sequence of vanishingly small perturbations of the Nash equilibrium strategies that has as its limit the equilibrium strategies. Kohlberg and Mertens require stability against <u>all</u> perturbations from a given class.

CHAPTER 3

MULTI-PERIOD AGENCY MODEL WITH AUDITOR REPUTATION

The purpose of this chapter is to define and analyze a multi-period game among the owner, manager and auditor. The auditor's role is to produce information to improve contracting between the owner and manager. In order to understand the demand for auditing we start by considering a standard principal-agent model.

In Section 2 we introduce a multi-period model of an owner-manager firm. The owner chooses the manager's contract while the manager provides a productive effort and issues a report. We need only consider pure strategy equilibria in which the manager reports "truthfully". Using the concept of sequential equilibrium we show that the optimal contract is one that pays the manager a constant almost everywhere in each period. Although such a contract imposes no risk on the manager it does not provide him with any incentive to take any action other than the minimum level of effort. It is this incentive problem that provides a demand for audit information.

In Sections 3 and 4 we add the auditor to the owner-manager game. The auditor exerts effort to monitor the manager's report and issues his own report. The owner uses the audit report to provide incentives for the manager. We assume the auditor to be a rational economic agent who acts in his own self-interest and has disutility for effort. Reputation considerations on the part of the auditor induce the manager and auditor to work hard and report truthfully. In Section 5 we amend and extend the game of Section 4 to allow for imperfect auditing and derive a sequential equilibrium of the amended game. Comparative static properties of the equilibrium are provided in Section 6. In Section 7 we examine various definitions of auditor independence within the context of our model.

3.2. MULTI-PERIOD OWNER-MANAGER MODEL

The owner of a production process hires a manager to perform duties that affect the outcome of that process. We consider a T period model. In each period t the manager is to take some action $a_t \in A_t \subseteq R$. We assume that there exist production functions $x_t = x_t(a_t, \omega_t) \in X \subseteq R$ and state spaces ω_t with densities $f(\omega_t)$, $t=1,\ldots,T$, which satisfy $f(\omega_1, \omega_2, \ldots, \omega_T) = f(\omega_1)f(\omega_2)\ldots f(\omega_T)$.

Only the manager observes the realization of x_t. After observing x_t, the manager announces the purported value of x_t, called $\hat{x}_t \in X$. Let F_X be some σ-algebra of subsets of X and $\mathcal{X}$ be the space of all F_X-measurable functions from X to X. The manager's reporting decision is the selection of a function denoted $\hat{x}_t(.)$, from $\mathcal{X}$. x_t and $\hat{x}_t(x_t)$ are both elements of X. The manager's report $\hat{x}_t$ is common knowledge.

The space of pure strategies for the owner is the set of all F_X-measurable functions from X to R or a subset of R. We denote this function space by S and call its elements, s, manager's contracts. The value $s_t(\hat{x}_t)$ represents cash transferred from the owner to the manager at time t. The owner's utility increases in $x_t(a_t, \omega_t) - s_t(\hat{x}_t(x_t(a_t, \omega_t)))$. The manager receives $s_t[\hat{x}_t(x_t(a_t, \omega_t))]$.[1]

The space of moves for the manager at each of his information sets is $A \times X$. The space of his pure strategies is the space of all appropriately measurable functions from S to $A \times X$.

The principal and the agent are assumed to possess temporal von Neumann-Morgenstern utility functions additively separable by periods. The principal's utility function is $G = \sum_{t=1}^{T} G_t[x_t - s_t]$ and the agent's is $U = \sum_{t=1}^{T}[W_t(s_t) - V_t(a_t)]$.

$$G_t'(\bullet) > 0, \quad G_t''(\bullet) \le 0$$

$$W_t'(\bullet) > 0, \quad W_t''(\bullet) < 0$$

$$V_t'(\bullet) > 0, \quad V_t''(\bullet) > 0$$

Following Holmstrom (1979), the cash flows in each period can also be represented as a random variable whose distribution is parameterized by the agent's actions. The outcome distributions are assumed to possess the following properties $\forall (a_1, a_2, \ldots, a_T)$, where F is a symbol for a cumulative distribution function and f is a symbol for the probability density function.

$$F(x_1, x_2, \ldots x_T | a_1, \ldots, a_T) = F(x_1|a_1) \ldots F(x_T|a_T) \tag{2.1}$$

$$F_{a_t}(x_t|a_t) \le 0 \text{ (with } < 0 \text{ for some } x_t) \; \forall \; t \tag{2.2}$$

$$F_{a_\tau}(x_t|a_t) = 0 \; \forall \; x_t \quad \text{when} \quad \tau \ne t \tag{2.3}$$

Condition (2.1) implies that the state variables are independently distributed over time. Condition (2.2) implies that the distribution of each period's cash flow shifts to the right in the sense of first order stochastic dominance with increased effort in that period. Condition (2.3) means that effort in one period has no effect on the outcome in any other period.

The supports of the outcome distributions are assumed to be independent of the agent's effort. Recall that the outcome $x_t(\cdot,\cdot)$ is not observable to the principal. Contracting must be confined to $\hat{x}_t(x_t)$, the agent's announcement of $x_t(\cdot)$. The principal knows the set of actions available to the agent and the agent's utility function. The set of feasible actions and the set of feasible sharing rules in one period do not depend on anything that occurs earlier in the game. The agent is assumed to have an outside employment opportunity with a utility level of O_t in period t.

The solution concept we adopt is Kreps and Wilson's (1982b) sequential equilibrium. They define an *assessment* to be composed of a system of beliefs and a strategy for each player. A system of beliefs specifies where each player believes he is in the game when it is his turn to move and he has received a particular information signal. A strategy specifies how each player will act at each point in time, conditional upon the information he has at that time.

We first consider a repetition of a one period game over time. At the start of each period, the principal (owner) announces a compensation scheme for the agent and the agent responds by choosing how much effort to supply in that period. The following time line illustrates the sequence of choices in the model.

Principal chooses s_1	Agent chooses $(a_1,\hat{x}_1(\bullet))$	x_1 realized	Agent reports, Principal observes $\hat{x}_1(x_1)$	Agent paid $s_1(\hat{x}_1(\bullet))$	Principal chooses s_2

Let $a_t \equiv (a_1,\ldots,a_t)$ and $\xi_t \equiv (s_1,\ldots,s_t)$ be the history of actions and sharing rules in periods $1,\ldots,t$. Let $X_t = (x_1,\ldots,x_t)$ be the sequence of cash flows observed by the agent. Let $x_t = (\hat{x}_t(\bullet),\ldots\hat{x}_t(\bullet))$ be the history of reporting functions chosen by the agent in periods $1,\ldots,t$. After the principal selects the sharing rule, he will observe $\hat{X}_t = (\hat{x}_1(x_1),\ldots\hat{x}_t(x_t))$. At the time the principal selects the sharing rule in period t, he will have observed $y_{pt} = (\hat{X}_{t-1},\xi_{t-1}) \in \eta_{pt}$ where η_{pt} is the set of possible histories of reported outcomes and sharing rules through period t-1. When the agent chooses $(a_t,\hat{x}_t(\bullet))$ in period t, he will have observed $y_{at} = (X_{t-1},\hat{X}_{t-1},\xi_t,a_{t-1}) \in \eta_{at}$ where η_{at} is the set of possible histories of cash flows and actions through period t - 1 and the sharing rules through period t.

A strategy for the principal is defined as a sequence of measurable functions mapping each information signal that he could receive into a sharing rule. Let $\underline{s} = [s_t(y_{pt},\hat{x}_t(x_t))] \; \forall \; y_{pt}$ and $\forall$ t denote a strategy for the principal. Similarly, a strategy for the agent is a sequence of functions $\underline{a} = [a_t(y_{at}),\hat{x}_t(y_{at})] \; \forall \; y_{at}$ and $\forall$ t.

Let $\kappa(a_t, X_t | \hat{X}_t, \xi_t)$ be the function which specifies the principal's beliefs concerning what actions the agent has taken and the actual outcomes observed by the agent in the first t periods, based on the sharing rules that were offered and the reported cash flows in the first t periods.

Since the utility functions and the production functions are assumed to be separable over time, once period t is reached, the expected utilities of the principal and agent in period t, depend only upon the sharing rule chosen by the principal, and action and reporting function chosen by the agent.

Define

$$EG_t[s_t, a_t, \hat{x}_t] = \int G_t[x_t - s_t[\hat{x}_t(\bullet)]]f(x_t|a_t)dx_t$$

$$EU_t[s_t, a_t, \hat{x}_t] = \int W_t[s_t[\hat{x}_t(\bullet)]]f(x_t|a_t)dx_t - V_t(a_t)$$

to be the expected utilities of the principal and agent respectively, when sharing rule s_t, action a_t and reporting function $\hat{x}_t(\bullet)$ are chosen.

Then $EG_t[s_t(y_{pt}, \hat{x}_t(x_t)), [a_t(y_{at}), \hat{x}_t(y_{at})]]$ and $EU_t[s_t(y_{pt}, \hat{x}_t(x_t)), [a_t(y_{at}), \hat{x}_t(y_{at})]]$ denotes the expected utilities of the principal and the agent respectively in period t, conditional upon y_{pt} being observed by the principal and y_{at} being observed by the agent when strategy $(\underline{s}, \underline{a})$ is used. Using the system of beliefs, we can define their expected utilities in period t, as assessed from the beginning of period j, by integrating over the possible realizations of the signals y_{pt} and y_{at}

that might occur in period t, given that their information in period j is y_{pj} and y_{aj}, $j \le t$. These expected utilities are given by $E[EG_t[s_t(y_{pt},\hat{x}_t), [a_t(y_{at}),\hat{x}_t(y_{at})]] \mid y_{pj}]$ and $E[EU_t[s_t(y_{pt},\hat{x}_t), [a_t(y_{at})\hat{x}_t(y_{at})]] \mid y_{aj}]$ for the principal and agent respectively. We further allow the agent's utility for money to be unbounded below. This condition ensures that the agent receives exactly O_t, his minimum utility level, in the tth period. (See Lambert (1981) for an excellent discussion on this point).

In the game outlined, an assessment is sequentially rational if (i) $\forall$ j and $\forall$ y_{pj}:

$$E[\sum_{t=j}^{T} EG_t[s_t(y_{pt},\hat{x}_t),[a_t(y_{at}),\hat{x}_t(y_{at})] \mid y_{pj}]$$

$$\ge E[\sum_{t=j}^{T} EG_t[s_t'(y_{pt},\hat{x}_t),[a_t(y_{at}),\hat{x}_t(y_{at})]] \mid y_{pj}] \qquad (2.5a)$$

$\forall$ other strategies $\underline{s}' = [s_t']$ for the principal and (ii) $\forall_j$ and $\forall$ y_{aj}

$$E[\sum_{t=j}^{T} EU_t[s_t(y_{pt},\hat{x}_t),[a_t(y_{at}),\hat{x}_t(y_{at})]] \mid y_{aj}]$$

$$\ge E[\sum_{t=j}^{T} EU_t[s_t(y_{pt},\hat{x}_t'),[a_t'(y_{at}),\hat{x}_t'(y_{at})]] \mid y_{aj}] \qquad (2.5b)$$

$\forall$ other strategies $\underline{a}' = [a_t',\hat{x}_t']$ for the agent.

We define id^X to be the identity function from X to X. We say a manager's contract s is *truth-inducing* when (a_t^*, id_t^X), for some a_t^*, maximizes $EU_t[s_t(y_{pt},\hat{x}_t),[a_t(y_{at}),\hat{x}_t(y_{at})]]$ over A x X.

PROPOSITION 3.2.1. If the strategies of the principal and the agent are to satisfy (2.5a) and (2.5b), the sharing rule and action in each period will be selected such that they are the solution to the problem (2.6) below, and the only feasible contracts are those equal to a constant almost everywhere (a.e.) in each period.[2]

$$\max_{s_t(\hat{x}_t),a_t} \int G_t[x_t - s_t(\hat{x}_t(\bullet))]f(x_t|a_t)dx_t \qquad (2.6A)$$

subject to

$$\int W_t[s_t(\hat{x}_t(\bullet))]f(x_t|a_t)dx_t - V_t(a_t) \geq 0_t \qquad (2.6(i))$$

$$(a_t,\hat{x}_t) \in \underset{A \times X}{\operatorname{argmax}} \int W_t[s_t(\hat{x}_t(\bullet))]f(x_t|a_t)dx_t - V_t(a_t) \qquad (2.6(ii))$$

PROOF: See Appendix to Chapter 3.

Consequently, the constrained optimal contract imposes no risk on the manager and provides no incentives. This arises because there are no opportunities to contract on any variable that is not wholly controlled by the manager. The separability of the utility functions and the production functions over time and the fact that the outcome in one period does not affect the utility function and the production function in any future period drives this result.

Sequential rationality requires that given any history of ξ_t and $\hat{X}_t$, the entrepreneur uses the belief function $\kappa(a,X_t|\hat{X}_t,\xi_t)$ to determine his expected utility maximizing choice of sharing rule (and the corresponding choice of action by the agent). However, as indicated in the proof of Proposition 3.2.1, we assume that if the agent is indifferent over a set of actions, he will choose the action from the set which maximizes the principal's expected utility. Furthermore the optimal (t+1)st period action will only depend on the (t+1)st period sharing rule. Consequently, the result does not depend upon the form of the principal's beliefs concerning the prior actions and choice of reporting rules of the agent.

It is also fairly evident that the same problems remain if both principal and agent precommit to a T period contract at the beginning of the first period. The problem is that since x_t and $\hat{x}_t$ are both "controlled" by the agent, it is impossible to condition later period contracts on prior period's reported figures, $\hat{x}_t$.

PROPOSITION 3.2.2: Under a contract with precommitments, the only feasible contracts are those equal to a constant almost everywhere in each period.

PROOF: See Appendix 3.

We consider the introduction of a second agent, the auditor, to produce mutually observable reports.[3] However, we also allow the auditor to be subject to moral hazard. This has generated discussion on who audits the auditor. We propose to argue that reputation formation may be rational economic behavior for the auditor. This would mitigate, though not eliminate, the problem of moral hazard on the auditor. The owner is able to condition payments to the manager on reported cash flows and still have the manager report the actual cash flows. Such a contract will provide incentives for the manager to take acts that are more productive than the acts chosen by the manager when no auditor is present and a constant amount is paid. The difference in payoffs to the owner from forcing this act versus paying the manager a fixed amount and not employing the auditor provides an upper bound on the amount the owner will be willing to spend to hire an auditor. For the rest of this paper we assume that audit costs are less than this upper bound. The fee paid to the auditor is such that it guarantees the auditor his minimum utility from alternative employment so that the auditor is no worse off by accepting the audit. If the moral hazard problem on the auditor is mitigated, hiring the auditor can be a Pareto efficient move in that it makes the owner better off without making the manager or auditor any worse off.

Before proceeding to the next section, we clarify our concerns about truthful reporting by the manager and auditor. Recall that in the equilibrium of Propositions 3.2.1 and 3.2.2, the manager did report truthfully. However, a necessary condition to achieve this was that the owner commit to disregard the report by making the manager's payment independent of the manager's report. As is well known, however, such a contract does not motivate the manager to take the act desired by the principal. Our concern in the subsequent sections of this chapter is to examine conditions under which the manager and auditor report truthfully even though they know full well the owner will use the reports against them. We show that in equilibrium, the principal writes a contract contingent on the manager's and auditor's reports and that the manager and auditor find it in their interest to report the outcome truthfully.

3.3 MULTI-PERIOD OWNER MANAGER AUDITOR MODEL

To the structure of the owner-manager model, we add a set C of possible actions for the auditor. C might represent possible levels of effort for the auditor. We refer to $c \in C$ as the auditor's investigative act.

The auditor's investigative act also induces a variable ζ that takes values in some abstract space Γ. ζ can be interpreted as some information about the auditor's investigative act and represents the auditor being sued by the owner for shirking, or as a result of bankruptcy or a bank investigation. ζ can also be interpreted as some form of peer review. The interpretation we focus on however, is that ζ is the report obtained from the court when the principal sues the auditor. ζ is a function of (c,x).

Strategy Spaces

To allow for the maximum moral hazard on the part of the auditor we allow the auditor to observe the cash flow (rather than some signal correlated with cash flow) with a probability that increases with higher levels of effort.[4] The auditor chooses a function $\hat{n}(\bullet)$ from X, the space of all F_X-measurable functions from X to X. $\hat{n}$ can be interpreted as the audited reported cash flow datum. The value $\hat{n}$ is common knowledge, so the manager's contracts may now depend on $\hat{x}$ and $\hat{n}$. Therefore, we define S to be the space of all measurable functions from X x X to R. Cash transfers from the owner to the manager are now denoted $s(\hat{n},\hat{x})$.

The owner also chooses a scheme of transfers from himself to the auditor. The auditor is assumed to be risk averse. If the owner is risk neutral and if moral hazard problems on the auditor are absent, it would be efficient to pay the auditor a fixed fee g for a level of audit c. This would also capture the idea of financial independence alluded to in the popular press that payments to the auditor are unrelated to the cash outcomes of the *firm*. ζ, the information revealed in the event of litigation, plays a role in this scheme. The owner has the option of committing to purchase ζ for $k \geq 0$ dollars with any probability θ that may depend on $\hat{x}$ and $\hat{n}$. If ζ is purchased, the auditor's payments can depend on $\hat{x}$, $\hat{n}$ and ζ, denoted by $g^2(\hat{x},\hat{n},\zeta)$. If ζ is not purchased, the auditor's payments depend on $\hat{x}$, $\hat{n}$ denoted by $g^1(\hat{x},\hat{n})$.[5]

The auditor is assumed to possess a temporal von Neumann-Morgenstern utility function additively separable by periods. The auditor's utility function is

$$H = \sum_{t=1}^{T} H_t = \sum_{t=1}^{T} \left[Q_t(g_t(\bullet)) - R_t(c_t)\right]$$

$$Q_t'(\bullet) > 0, \qquad Q_t''(\bullet) < 0$$

$$R_t'(\bullet) > 0, \qquad R_t''(\bullet) > 0$$

where $Q_t(\bullet)$ denotes utility to the auditor for cash and $R_t(\bullet)$ denotes the non-pecuniary return in period t.

Define

$$EG_t[(s_t, g_t, \theta_t), (a_t, \hat{x}_t(\bullet)), (c_t, \hat{n}_t(\bullet))]$$

$$= \int_x [(1 - \theta_t(\hat{n}_t, \hat{x}_t))G_t[x_t - s_t(\hat{n}_t, \hat{x}_t) - g_t^1(\hat{x}_t, \hat{n}_t)]f(x_t|a_t)dx_t$$

$$+ \theta_t(\hat{n}_t, \hat{x}_t)G_t[x_t - s_t(\hat{n}_t, \hat{x}_t) - g_t^2(\hat{x}_t, \hat{n}_t, \zeta_t) - k]f(x_t|a_t)dx_t$$

$$EU_t[(s_t, g_t, \theta_t), (a_t, \hat{x}_t(\bullet)), (c_t, \hat{n}_t(\bullet))] = \int W_t(s_t(\hat{x}_t, \hat{n}_t))f(x_t|a_t)dx_t - V_t(a_t)$$

to be the expected utilities of the principal and agent respectively.

This reflects the fact that if $\zeta_t(\bullet)$ is purchased (with probability θ_t), after incurring investigation costs of k, the principal conditions payments to the auditor on $\zeta_t(\bullet)$.[6] If $\zeta_t(\bullet)$ is not purchased, the principal makes payments of $s_t(\bullet)$ and $g_t^1(\bullet)$ to the manager and auditor respectively, based only on the manager's and auditor's reports.

When the principal chooses the manager's sharing rule, the audit fees and investigation strategy in period t, he will have observed y_{pt}, where

$$y_{pt} = (\hat{X}_{t-1}, \hat{N}_{t-1}, \xi_{t-1}, Z_{t-1}, \mathbf{g}_{t-1}) \quad \text{where} \quad \hat{N}_t = (\hat{n}_1 \ldots, \hat{n}_t)$$

$$Z_t = (\zeta_1, \ldots, \zeta_t), \ \mathbf{g}_t = (g_1, \ldots, g_t), \ \text{and} \ g_t = \begin{cases} g_t^1 & \text{if } \xi_t \text{ is not purchased} \\ g_t^2 & \text{if } \xi_t \text{ is purchased} \end{cases}$$

We adopt the convention that $\zeta_t = 0$ if the principal does not investigate in period t. When the manager chooses $(a_t, \hat{x}_t(\bullet))$ in period t, he will have observed y_{at} where

$$y_{at} = (X_{t-1}, \xi_t, a_{t-1}, C_{t-1}, g_t, \hat{N}_{t-1}, \hat{X}_{t-1}, Z_{t-1}) \text{ where } C_t = (c_1, \ldots, c_t)$$

When the auditor chooses $(c_t, \hat{n}_t(\bullet))$ in period t, he will have observed y_{ht} where

$$y_{ht} = (X_{t-1}, \hat{X}_t, \xi_t, g_t, C_{t-1}, \hat{N}_{t-1}, Z_{t-1}).$$ [7]

A strategy for the auditor is defined as a sequence of measurable functions mapping each information signal that the auditor could receive into an action choice and reporting rule. Let $\underline{c} = (c_t(y_{ht}), \hat{n}_t(y_{ht})) \ \forall \ y_{ht}$ and $\forall$ t.

Define,

$$EH_t\left[(s_t, g_t, \theta_t), (a_t, \hat{x}_t(\bullet)), (c_t, \hat{n}_t(\bullet))\right] =$$

$$\int_X \Big[(1-\theta_t(y_{pt}, \hat{x}_t, \hat{n}_t))\, Q_t(g_t^1(\hat{x}_t, \hat{n}_t))$$

$$+ \theta_t(y_{pt}, \hat{x}_t, \hat{n}_t) Q_t(g_t^2(\hat{x}_t, \zeta_t, \hat{n}_t))\Big] f(x_t | a_t) dx_t - R_t(c_t)$$

to be the auditor's expected utility in period t.

In the game outlined, an assessment is sequentially rational if $\forall$ j and $\forall$ y_{pj}, y_{aj} and y_{hj}

(i)
$$E\Big[\sum_{t=j}^{T} EG_t\big[s_t(y_{pt},\hat{x}_t,\hat{n}_t),g_t(y_{pt},\hat{x}_t,\hat{n}_t),\theta_t(y_{pt},\hat{x}_t,\hat{n}_t),$$
$$(c_t(y_{ht}),\hat{n}_t(y_{ht})),(a_t(y_{at}),\hat{x}_t(y_{at}))\big] \mid y_{pj}\Big] \geq$$
$$E\Big[\sum_{t=j}^{T} EG_t\big[s'_t(y_{pt},\hat{x}_t,\hat{n}_t),g'_t(y_{pt},\hat{x}_t,\hat{n}_t),\theta'_t(y_{pt},\hat{x}_t,\hat{n}_t),$$
$$(c_t(y_{ht}),\hat{n}_t(y_{ht})),(a_t(y_{at}),\hat{x}_t(y_{at}))\big] \mid y_{pj}\Big] \tag{3.1}$$

$\forall$ other strategies $\underline{s}' = [s'_t,g'_t,\theta'_t]$ for the principal.

(ii)
$$E\Big[\sum_{t=j}^{T} EU_t\big[s_t(y_{pt},\hat{x}_t,\hat{n}_t),g_t(y_{pt},\hat{x}_t,\hat{n}_t),\theta_t(y_{pt},\hat{x}_t,\hat{n}_t),$$
$$(c_t(y_{ht})\hat{n}_t(y_{ht})),(a_t(y_{at}),\hat{x}_t(y_{at}))\big] \mid y_{aj}\Big] \geq$$
$$E\Big[\sum_{t=j}^{T} EU_t\big[s_t(y_{pt},\hat{x}'_t,\hat{n}_t),g_t(y_{pt},\hat{x}'_t,\hat{n}_t),\theta_t(y_{pt},\hat{x}'_t,\hat{n}_t),$$
$$(c_t(y_{ht}),\hat{n}_t(y_{ht})),(a'_t(y_{at}),\hat{x}'_t(y_{at}))\big] \mid y_{aj}\Big], \tag{3.2}$$

$\forall$ other strategies $\underline{a}' = [a'_t,\hat{x}'_t]$ for the agent.

$$\text{(iii)}\quad E\Big[\sum_{t=j}^{T} EH_t\big[s_t(y_{pt},\hat{x}_t,\hat{n}_t),g_t(y_{pt},\hat{x}_t,\hat{n}_t),\theta_t(y_{pt},\hat{x}_t,n_t),$$

$$(c_t(y_{ht}),\hat{n}_t(y_{ht})),(a_t(y_{at}),\hat{x}_t(y_{at}))\big] \mid y_{hj}\Big] \geq$$

$$E\Big[\sum_{t=j}^{T} EH_t\big[s_t(y_{pt},\hat{x}_t,\hat{n}_t'),g_t(y_{pt},\hat{x}_t,\hat{n}_t'),\theta_t(y_{pt},\hat{x}_t,\hat{n}_t'),$$

$$(c_t'(y_{ht}),\hat{n}_t'(y_{ht})),(a_t(y_{at}),\hat{x}_t(y_{at}))\big] \mid y_{hj}\Big] \qquad (3.3)$$

$\forall$ other strategies $\underset{\sim}{c}' = [c_t', \hat{n}_t']$ for the auditor.

We illustrate the requirements of sequential rationality using a two period model. Let $\kappa(a_1,c_1,x_1|\hat{x}_1,\hat{n}_1,s_1,g_1,\zeta_1)$ denote the principal's beliefs concerning the actions the agent and auditor have taken and the actual outcome in the first period. Sequential rationality requires that in the second period each player maximizes his expected utility taking beliefs as fixed.

(i) For each y_{a2}, the agent's strategy is such that

$$[a_2(y_{a2}),\hat{x}_2(y_{a2})] \in \text{argmax} \int W_2(s_2(\hat{x}_2,\hat{n}_2))f(x_2|a_2)dx_2 - V_2(a_2)$$

(ii) $s_2(y_{p2},\hat{x}_2,\hat{n}_2),g_2^1(y_{p2},\hat{x}_2,\hat{n}_2),g_2^2(y_{p2},\hat{x}_2,\hat{n}_2,\zeta_2) \in \text{argmax} \int \kappa(a_1,c_1,x_1|y_{p2})$

$$\cdot\ \{[(1-\theta_2(y_{p2},\hat{x}_2,\hat{n}_2))G_2(x_2-s_2(y_{p2},\hat{x}_2,\hat{n}_2) - g_2^1(y_{p2},\hat{x}_2,\hat{n}_2))$$

$$+\ \theta_2(y_{p2},\hat{x}_2,\hat{n}_2)\ G_2(x_2-s_2(y_{p2},\hat{x}_2,\hat{n}_2,\zeta_2) - g_2^2(y_{p2},\hat{x}_2,\hat{n}_2,\zeta_2) - k)]$$

$$f(x_2|a_2(y_{a2}))dx_2\}da_1dc_1dx_1$$

(iii) For each y_{h2}, the auditor's strategy is such that

$$[c_2(y_{h2}),\hat{n}_2(y_{h2})] \in \operatorname{argmax} \int \{(1-\theta_2(y_{p2},\hat{x}_2,\hat{n}_2))Q_2(g_2^1(\hat{x}_2,\hat{n}_2))$$

$$+ \theta_2(y_{p2},\hat{x}_2,\hat{n}_2)Q_2(g_2^2(\hat{x}_2,\hat{n}_2,\zeta_2))f(x_2|a_2(y_{a2}))\}dx_2 - R_2(c_2)$$

We have seen that the equilibrium in the owner-manager game was for a constant amount to be paid to the manager. The question we seek to address here is whether the introduction of the auditor allows the owner to create incentives for the manager to take productive acts that increase the owner's expected utility. The owner could achieve this by making the manager's compensation contingent on the outcomes reported by the manager and auditor and yet have the manager and auditor report the outcome truthfully.

This is a somewhat delicate issue. The owner will attempt to maximize his expected utility subject to the auditor and manager playing a sequentially rational equilibrium in the subgame between the two. But, in general, all equilibria of a two person game do not provide the players with the same expected utilities. We are not guaranteed that the subgame equilibrium that maximizes the owner's expected utility also maximizes the auditors and manager's expected utilities. Therefore, there could exist subgame dominant equilibria which do not maximize the owner's expected utility over all the equilibria of the subgame. One classic instance of this is for both the manager and auditor to shirk, to the detriment of the owner. We elaborate on these concerns by considering a simple game with binary state spaces (good outcome/bad outcome) and binary action choices (work/shirk) for the manager and auditor.

3.4. SEQUENTIAL EQUILIBRIUM OF THE OWNER-MANAGER-AUDITOR MODEL

We assume the game evolves dynamically over time, i.e., the owner specifies the game to be played in period t+1 given the history y_{pt} up to period t. The game is played over a finite number of periods T. We assume at the start of the game the owner and manager precommit to an S period contract where $S \leq T$. For simplicity in notation and without loss of generality, we assume $S = T$ ($S < T$ can be interpreted as different S period contracts being written with managers over the T period horizon. The length of S influences the optimal contract but is not critical to the analysis). The principal first contracts with the auditor who then chooses his act. The contract specifies conditions under which the principal may choose to incur a further cost k to investigate the auditor.

We know that in the two outcome case with outcomes observable, the payoff to the manager s_g when a good outcome occurs will be greater than the payoff s_b when a bad outcome occurs (Grossman and Hart, 1982). However in the model described here x is not observable and contracting must be confined to $\hat{x}$, the reported outcome. We analyze conditions under which, in equilibrium, the manager correctly reports x, despite being faced with a contingent contract on the reported cash flow. $s(\hat{x})$ is then an increasing function of the reported outcome $\hat{x}$.

3.4.1 DESCRIPTION OF STRATEGIES AND PAYOFFS

The probabilities of a good or bad outcome are parameterized by the action a_t taken by the manager. p_t^1 denotes the probability of a good outcome occurring in period t should the manager shirk in his choice of act (i.e., choose act a_t^1 rather than the optimal act a_t^* desired by the principal). p_t^* denotes the probability of a good outcome occurring should the manager work hard (i.e., choose the act a_t^* desired by the principal). Of course, $p_t^1 < p_t^*$.

Let b_t^{wg} (b_t^{wb}) represent the manager's expected utility in period t if he works hard, a good (bad) outcome results and the manager reports the outcome truthfully. Let b_t^{sg} (b_t^{sb}) represent the manager's expected utility in period t if he shirks, a good (bad) outcome occurs and the manager reports the outcome truthfully. b_t^{sg} also represents the manager's expected utility in period t if he shirks, a bad outcome results which the manager reports as good and the misreporting is not detected. Let $b_t^{sg} - \beta_t$ denote the manager's expected utility if he shirks, reports a bad outcome as good and is detected by the auditor.

The owner precommits to an S period contract with the manager on the basis of reported and audited outcomes. (We could model the manager as precommitting as well; alternatively, we could give the manager the right to leave at the end of each period. See, for instance, Lambert (1981). Whereas the contract offered is different in the two cases, it does not affect our main conclusions.) The agency contract is so structured that the manager gets a minimum utility level of $\sum_{t=1}^{T} O_t$ if the manager takes the desired action each period and reports truthfully. The agency contract also guarantees the auditor his minimum utility level each period.

We later show that the auditor's incentives are so designed by the principal that the manager's optimal strategy is to report the outcomes truthfully over a large number of periods in the game, even though, unlike the truth inducing mechanisms of the owner-manager game, the manager's payments in the owner-manager-auditor game are contingent on the manager's reports.

The strategy for the owner is to write contracts on reported outcomes which, in equilibrium, for a large number of periods are the actual outcomes that result from the manager's choice of action. The contract is structured so that the payments to the manager (and the optimal acts chosen by the manager) in future periods are a function of the history of prior period's outcomes. This can be interpreted as the owner using subsequent period contracts as a means of motivating the manager's effort more efficiently in earlier periods.

We next briefly describe some of the salient features of the contract. We will find it convenient to label time backwards, so that the first time period is denoted T and the last time period is denoted 1. a_T^* denotes the manager's optimal act in the first period of the T period contract. After act a_T^* is chosen, either a good outcome or bad outcome will result. The optimal act a_{T-1}^* for the manager in the second period is a function of the outcome in the previous period (since the contract offered in the second period depends on the first period's outcome). The manager's expected utility in period T-1 is also a function of the outcome in period T. The penalty $\beta_{T-1}(\cdot)$ in period

T-1 if the manager is caught bluffing by the auditor is also a function of the previous period's reported outcome. This simply reflects the fact that the penalty in period T-1 is an increasing function of the expected utility to the manager from shirking and bluffing and not being detected.

Thus, at each period t of the game the manager's sharing rule and action depend on the history of prior period's manager and auditor reports. A strategy available to the manager is to falsify his report. At each stage t the manager has two options, (i) he could report truthfully and take the optimal act desired by the principal or (ii) he could choose to misreport and choose the act a_t that maximizes his expected utility for the remainder of the game. The exact choice of act in (ii) above depends on the nature of the production function. For instance, the production function may be such that the manager never shirks in his choice of act but chooses to bluff if a bad outcome arises. We accentuate the moral hazard problem on the manager by assuming that if misreporting is optimal for the manager, he would also find it in his interest to shirk in his choice of act. The manager's expected utility when he shirks and misreports, and the auditor works hard and detects his bluff is given by $p_t^1 b_t^{sg} + (1 - p_t^1)(b_t^{sg} - \beta_t)$. The manager's expected utility from working hard and reporting truthfully is $p_t^* b_t^{wg} + (1-p_t^*)b_t^{wb}$.

The optimal contract designed by the owner has the property that the expected utility to the manager from shirking (that is, taking an act lower than the desired act) and reporting truthfully, $p_t^1 b_t^{sg} + (1-p_t^1)\, b_t^{sb}$ is not greater than the expected utility to the manager from working hard and reporting truthfully. Since we are assuming only two actions, this constraint is binding. Thus $p_t^* b_t^{wg} + (1-p_t^*) b_t^{wb} = p_t^1 b_t^{sg} + (1-p_t^1) b_t^{sb}$. We then invoke the standard argument that since the manager has no incentive to deviate, he chooses the act a_t^* desired by the owner. Therefore, under this scheme, if the manager reports truthfully, he chooses the optimal act.

The penalty $\beta_t(y_{pt}, \hat{x}_t, \hat{n}_t)$ need not be very large. For each t, $\beta_t(y_{pt}, \hat{x}_t, \hat{n}_t)$ need only satisfy the property $b_t^{sg}(\bullet) - b_t^{sb}(\bullet) < \beta_t(\bullet)$. This reflects the fact that the expected utility to the manager from reporting a bad outcome as good and being detected is less than the expected utility to the manager from reporting a bad outcome as bad. Without this condition it would always pay the manager to shirk and bluff.[8] For all time periods t, we have $b_t^{sg} > p_t^* b_t^{wg} + (1-p_t^*) b_t^{wb} > p_t^1 b_t^{sg} + (1-p_t^1)(b_t^{sg} - \beta_t)$.

The owner contracts with the auditor to pay a fee g_t which is a function of the history y_{pt}. The expected utility to the auditor from working hard is $Q_t(g_t) - R_t(c_t^*) = m_t$. The utility to the auditor from shirking if he is not detected is $Q_t(g_t) - R_t(c_t^1) = e_t$. The owner has the option of purchasing ζ_t (that is, investigating the auditor) with some probability, θ_t. The owner commits to a mixed strategy θ_t which is a function of y_{pt}, $\hat{x}_t$ and $\hat{n}_t$.[9] If ζ_t is purchased and the auditor shirks when the manager reports a bad outcome as good, the expected utility to the auditor is denoted d_t ($d_t < m_t < e_t$). d_t reflects both the penalties assessed by a court and loss in existing and future business. The expected utility to the auditor if the auditor shirks, the manager reports correctly and the owner investigates is denoted as ϵ_t. $\epsilon_t < p_t^* m_t + (1 - p_t^*)e_t$. A table of notation for Chapter 3 is given in Table 3.0. The normal form of the game to be played in each period based on the strategies and payoffs described above is shown in Table 3.1.

By the auditor working hard we mean both (i) the auditor supplying a high level of effort and (ii) the auditor reporting truthfully what he finds. This implies that the auditor has fully complied with generally accepted auditing practices. Any other behavior on the part of the auditor is classified as shirking. We interpret failure to supply the desired level of effort as negligence. Failure to report findings truthfully is interpreted as lack of independence. The case of collusion discussed in the introduction is one where the auditor supplies a high level of effort, detects the manager's misreport but agrees (perhaps for a side payment) not to report the breach to the owner.

We define $\alpha_t = \theta_t d_t + (1 - \theta_t)e_t$, $l_t = p_t^* m_t + (1 - p_t^*)e_t$ and $\phi_t = \theta_t \epsilon_t + (1-\theta_t)e_t$. If at the optimal θ_t, $\alpha_t > m_t$ (we assume that for such a θ_t, $(1-\theta_t)e_t + \theta_t \phi_t > l_t$), we have a unique equilibrium to the single play game which entails the manager bluffing and the auditor shirking.[10] The important issue that we wish to analyse is whether this equilibrium in the subgame continues to exist as the dynamic game is played over T periods.

3.4.2 AUDITOR TYPES AND OWNER'S INVESTIGATION STRATEGY

It is easy to show that if the exact payoffs in the game are ***common knowledge*** (see Aumann (1976)) to the manager and auditor and if

$\alpha_t > m_t$ for all t, the auditor's optimal strategy is to shirk in every stage of the T period game.[11] We refer to such an auditor as the *weak* auditor.

We interpret α_t as the auditor's minimum level of utility from alternative employment. Once the auditor shirks he reveals himself to the manager as a weak auditor. We later show, in the reputation equilibrium, that once the auditor is revealed to be of the weak type, the manager's optimal strategy is to shirk and bluff. The auditor's expected utility, when the manager's strategy is to shirk and bluff and the auditor's strategy is to shirk, is α_t. Consequently, once the auditor shirks (in order to benefit from the reputation he has developed) the auditor is no worse off by not continuing as the auditor and instead seeking alternative employment.

The common knowledge assumption that we have imposed throughout our analysis thus far is an extremely strong one. For instance it is assumed to be common knowledge that the expected utility m_t to the auditor from working hard is less than the expected utility α_t. This may not necessarily be the case. The auditor may benefit from positive information externalities by working hard. Working hard may enable the auditor to produce private information to develop an analytic review base or generate some kind of learning.[12]

Auditors may also differ in terms of their perceived "costliness" of penalties in the event of any shirking on the auditor's part being detected. It is quite conceivable the expected disutility of the penalty is large for some auditors whereas others may believe the probability of detection and the risk to be rather minimal. The latter type of auditors will shirk, whereas the former will prefer to work.

The important point here is that the audit firm is playing a bigger game (involving the audits of other corporations both in this period and in future periods) of which the game against any particular manager is only a part. Thus, it is difficult for the manager to be certain that what is optimal for the auditor in the part-game is optimal in terms of the auditor's overall strategy. When m_t is greater than α_t for all t, the optimal strategy for the auditor is to always work hard. We refer to such an auditor as the ***strong*** auditor.

It is easy to verify that a pure investigation strategy of never investigating or always investigating is not optimal for the principal. If the owner never investigates, the optimal strategy for the manager is to shirk and misreport and the optimal strategy for the weak auditor is to shirk. In such a case the owner is better off not employing the auditor at all and playing the owner-manager game discussed in the earlier section. Thus, the pure strategy of never investigating is not an equilibrium strategy for the owner in the owner-manager-auditor game.

Moreover, the pure strategy of always investigating in period t is also not an equilibrium strategy for the owner. If the owner always investigates, the manager takes the optimal act and reports correctly and the auditor works hard. Therefore, the owner has an incentive to deviate and not investigate, thereby saving the costly investigation cost k. The principal's optimal strategy is to commit to a mixed strategy and investigate with probability θ_t which is a function of the history $(y_{pt}, \hat{x}_t, \hat{n}_t)$. The optimal θ_t is characterized in the sequential equilibrium computed later.

In the next section, we characterize the sequential equilibrium for the game discussed above. Sequential equilibria are extremely difficult to calculate for even the most simple games. In fact, the exact sequential equilibrium strategies have not even been calculated for the finitely repeated prisoner's dilemma (see Kreps et al (1982)). We therefore compute the equilibrium for a simple game, and apply the intuition gained to more complex settings.

3.4.3 CHARACTERIZATION OF THE SEQUENTIAL EQUILIBRIUM

We start by reviewing briefly the strategic moves for each of the players. The strategic move for the owner is the design of the subgame to be played in terms of the manager and auditor contracts and the choice of when to investigate. For the auditor it is what level of effort to supply and the report $\hat{n}_t$ to be made. The manager must decide the level of effort to supply and the cash flow to report. We model the game as follows.

The auditor can be either of the two types discussed above. The auditor knows his type, but the manager is unsure and assesses some probability δ that the auditor is of the ***strong*** type and $1-\delta$ that the auditor is of the ***weak*** type.[13] In terms of our analysis it is not even necessary that the strong type auditor actually exist. All that is necessary is that the manager ***believe*** that the strong auditor exists and that the auditor and manager know this.

In this simple model we assume that if the auditor works hard he will be able to determine the true outcome perfectly. If the auditor shirks he simply accepts the report of the manager. We relax these assumptions later. As the game progresses, the manager observes all prior moves. Consequently, history of moves prior to stage t may enable the manager to revise his assessment, if the history reveals some information about the relative likelihoods of the auditor's type.

Sequential equilibrium requires the following: (a) Whenever a player chooses an action, that player has some probability assessment over the nodes in his information set, reflecting what the player believes has happened so far. (b) These assessments are consistent with the hypothesized equilibrium strategy satisfying Bayes' rule whenever it applies. (c) Starting from every information set (including those off the equilibrium path), the strategy used by the player whose turn it is to move, is optimal from then on.

Recall that the last period is called Period 1 of the game, the last but one stage, Period 2 and so on with the first stage being Period T. Define q_t to be the probability that the auditor is assessed to be of the strong type in period t. From our earlier description $q_T = \delta$. It will also be convenient to denote $b_t^{sg} - b_t^{sb}/\beta_t$ as b_t^1. Note that $b_t^1 < 1$.

The equilibrium given below is rather involved. Before writing down the equilibrium we give a brief idea of what the nature of the proof is. In the absence of any perceived possibility of a strong auditor (δ=0) the equilibrium in the game is for the weak auditor to shirk and the manager to bluff. However, since there is a small probability that the auditor facing the manager is of the strong type, it pays the weak auditor to imitate the strong one by working hard. In the sequential equilibrium we derive this is not an idle threat. Roughly, the argument goes as follows. For sufficiently early stages in the game such a strategy by the weak auditor would force the manager to tell the truth. The weak auditor would be happy to follow this strategy since his expected utility when the manager works hard and reports truthfully is greater than his expected utility when the manager shirks and bluffs. The strategies towards the end of the game are more complicated. It turns out to be optimal for the weak auditor to adopt a mixed strategy, shirking with some probability.

Proposition 3.4.1 describes the strategies that lead to the sequential equilibrium. Proposition 3.4.2 suggests that but for a few points, these strategies are essentially unique. We assume that the investigation cost k and the payoff d_t to the auditor if the auditor shirks and the owner investigates are such that for the weak auditor, $\alpha_t > m_t$ and $(l_t + p_t^*(\phi_t - m_t)) > l_t$, that is $\phi_t > m_t \forall t=1,\ldots T$.

We next give a sequential equilibrium for this game. Recall that we defined q_t as the probability the manager views the auditor he is facing as strong at stage t. $q_T = \delta$. For $t < T$ we have the following recursive definition of q_t. (i) If a bad outcome arises and the manager reports it as bad, the manager cannot update the probability of the auditor being strong, because both types of auditors simply accept the manager's report, so $q_t = q_{t+1}$. (ii) If a good outcome is reported and the auditor works hard, $q_t = \max(\Pi_{i=1}^{t} b_i^1, q_{t+1})$. (iii) If a good outcome is reported and the auditor shirks, $q_{t+1} = 0$. (iv) $q_t = 0$ for all $t = 1,2,\ldots T-1$ if $q_{t+1} = 0$.

We next describe the strategies of the players at each stage in the game. It will be convenient to define $\tau(q) = \inf\left[t: \Pi_{i=1}^{t} b_i^1 < q \text{ for } q > 0\right]$

STRATEGIES OF THE OWNER (1) The owner's problem is to maximize expected utility subject to the optimal actions chosen by the manager and auditor in the subgame designed by the owner and described in Table 3.1. The owner contracts with the manager on the basis of the outcomes reported by the manager and auditor. The owner writes contingent contracts as if the outcomes reported by the manager and verified by the auditor are the actual outcomes. The contract that the owner writes with the manager is a function of the history $(y_{pt},\hat{x}_t,\hat{n}_t)$.

(2) The owner pays the auditor a fee g_t each period, and chooses an investigation strategy θ_t (which is a function of the history $(y_{pt},\hat{x}_t,\hat{n}_t)$) such that

$$\text{(i)}\quad m_t + \sum_{i=\tau(q)}^{t-1} l_i + p^*_{\tau(q)}[\phi_{\tau(q)} - m_{\tau(q)}] > \phi_t + \sum_{i=\tau(q)}^{t-1} \alpha_i \quad \forall\, t \geq \tau(q)$$

$$\text{(ii)}\quad \phi_t - m_t < l_{t-1} + p^*_{t-1}\,(\phi_{t-1} - m_{t-1}) - \alpha_{t-1} \qquad \forall\, t < \tau(q)$$

We refer to these inequalities as condition (A).[14,15]

The expected benefit to the owner of choosing a sufficiently high θ_t, such that $m_t > \alpha_t$, is that by forcing the auditor to work hard the manager would be forced to work hard and report truthfully. However, if the expected investigation cost each period $\theta_t k$ exceeds the expected benefit from forcing a higher managerial action, it would never be optimal to choose such a θ_t. Hence, we have α_t greater than m_t. In such a case, if the owner's investigation reveals that the auditor has shirked in a particular period, the owner may choose not to hire the auditor in subsequent periods. The game then reverts to the equilibrium without auditing described in Proposition 3.2.1. In the remaining plays of the game the owner pays the manager a constant wage and the manager supplies the minimum level of effort. The equilibrium we describe in Proposition 3.2.1 is consistent with the game evolving in this way.

STRATEGIES OF THE AUDITOR If a bad outcome is reported both types of auditors simply accept the report. If a good outcome is reported and (a) if the auditor is of the strong type, he always works hard as per the matrix of payoffs described earlier. (b) If the auditor is weak, he adopts the following strategies. If t = 1, he shirks. If t > 1 and

$q_t \geq \Pi_{i=1}^{t-1} b_i^1$, the weak auditor always works hard. If t > 1, and

$q_t < \Pi_{i=1}^{t-1} b_i^1$, the auditor works hard with probability

$$\frac{(1 - \Pi_{i=1}^{t-1} b_i^1)q_t}{(1-q_t)\Pi_{i=1}^{t-1} b_i^1} \text{ and shirks with probability } \frac{\Pi_{i=1}^{t-1} b_i^1 - q_t}{(1-q_t)\ \Pi_{i=1}^{t-1} b_i^1}.$$

STRATEGIES OF THE MANAGER

If $q_t > \Pi_{i=1}^{t} b_i^1$, the manager always reports truthfully and takes the action desired by the owner. If $q_t < \Pi_{i=1}^{t} b_i^1$, the manager shirks on the action choice and always reports a good outcome. If $q_t = \Pi_{i=1}^{t} b_i^1$, the manager randomizes, shirking and bluffing with probability $(\phi_t - m_t) / [l_{t-1}+p^*_{t-1}(\phi_{t-1}-m_{t-1}) - \alpha_{t-1}]$ if the manager had worked hard and reported truthfully in the previous period or (ii) shirking and bluffing with probability $\frac{(\alpha_t-m_t)}{[l_{t-1}+p^*_{t-1}(\phi_{t-1}-m_{t-1}) - \alpha_{t-1}]}$ if the manager had shirked and bluffed in the previous period.

It is worth noting that once the auditor shirks and captures the benefit from the reputation he has developed, the auditor has no incentive to continue as an auditor. Shirking reveals the auditor to be of the weak type so that the manager's optimal strategy in the remaining stages of the game is to shirk and bluff. The auditor's utility in such a case is α_t. Since α_t is the auditor's minimum utility level from alternative employment, the auditor is as well off leaving the audit and instead seeking alternative employment. The game then reverts to the equilibrium without auditing described in Proposition 3.2.1. In the remaining periods of the game the owner pays the manager a constant wage and the manager supplies the minimum level of effort. The equilibrium described in Proposition 3.4.1 is consistent with the above interpretation.

PROPOSITION 3.4.1 The strategies and beliefs given above constitute a sequential equilibrium in the game among the owner, manager and auditor provided condition (A) is satisfied.[16] The sequential equilibrium has the property that the manager works hard and reports truthfully and the weak auditor works hard and reports accurately over all stages of the game for which $t > \tau(q)$.

Proof: See Appendix 3

The maximum payment g_t to the auditor in period t must satisfy condition (A) of Proposition 3.4.1. We are thus able to characterize the conditions that determine auditor fee schedules in our model.

It is possible to interpret the auditor's expected utility in Proposition 3.4.1, as the marginal expected utility to the auditor from auditing an additional client in period t. The disutility of the penalty as a result of the auditor being caught shirking, can be interpreted as the marginal disutility to the auditor from such an event. This interpretation allows us to examine the game from the auditor's point of view in terms of all clients in all time periods T. The probability of detection θ_t in period t can then be interpreted as detection by any of the auditor's clients in period t. Consequently, the expected cost of investigation to the owner is a decreasing function of the number of the auditor's clients. This, in our model, provides an incentive for the owner to hire a large audit firm.

If we further assume ***plausible beliefs*** (Kreps and Wilson (1982a)), namely, that if we see an auditor working, the manager does not revise downwards the probability the auditor is strong, we can show that the equilibrium described above is essentially unique.

PROPOSITION 3.4.2: If $\delta \neq \Pi_{i=1}^{t} b_i^1$ for $t \leq T$, every sequential equilibrium with plausible beliefs has on-the-equilibrium-path strategies described in Proposition 3.4.1 (that is, the nature of the equilibrium is unique).

Proof: See Appendix 3

Loosely speaking, we can interpret successive values of q_t as the "reputation" of the auditor for hard work and independence at time t. In our model, q_t acts as a deterrent to the manager. When the value of q_t is sufficiently high, the manager prefers to work hard and report the truth. He believes that if he acts otherwise, the auditor will detect his bluff.

There are important distinctions between the evolution of reputation in this model relative to the simultaneous play game discussed in Kreps and Wilson (1982a). First, the gain or loss of reputation in our model depends on a random move by Nature. Consider the case where the auditor shirks and the manager works hard and reports truthfully. If Nature's move results in a bad outcome, there is no loss of reputation. If a good outcome occurs, the auditor is revealed to be weak. Naturally, this has the effect of reducing the value of building a reputation relative to the case where shirking is conclusive evidence of a weak auditor. In Proposition 3.4.1, we showed that the strategies of the manager and owner depend on p_t^*. Surprisingly, the auditor's strategy is independent of p_t^*. The reason is that the value of q_t at which the manager is indifferent between working hard and reporting truthfully, and shirking and bluffing, is independent of p_t^*. Second, we consider a three person, dynamic, principal-agent game. The auditor's incentive to develop a reputation in the subgame depends on the principal's optimal choice of θ_t. In other words, the auditor only finds it worthwhile to develop a reputation, if building a reputation is in the best interest of the principal.

3.4.4 COLLUSION IN AUDITOR-MANAGER SUBGAME

The analysis can easily be extended to the case where the auditor and manager collude in concealing a breach. Agreeing to collude and not report a breach will expose the auditor to some penalty and reveal the auditor to be of the weak type. The optimal strategy for the manager thereafter will be to shirk and bluff and attempt to collude with the auditor at each stage of the game. Using arguments similar to those used in Proposition 3.4.1, it can be shown that if q_t is sufficiently high, the optimal strategy for the auditor is not to collude but to instead report any breach to the owner. The benefit to the auditor from doing so arises from the manager working hard and reporting truthfully in future periods rather than shirking, bluffing and trying to collude with the auditor. (The essential argument is that the principal's investigation decision θ_t and the penalty, if collusion is detected, are such that the auditor's expected utility to the end of the game from reporting a breach is greater than the expected utility to the auditor from colluding.) It therefore pays the auditor to maintain his reputation for hard work and independence.

In the model discussed above, we assumed that the production function was such that it paid the manger to shirk when the bluffing strategy was adopted. It is quite conceivable that the production function, the agency contract and the principal's investigation strategy and penalties are such that the optimal strategy for the manager is to work hard, but to report the outcome to be good if a bad outcome should occur. In such a case, the same basic structure for the equilibrium emerges in that for sufficiently large t (that is, $q_t > \Pi_{i=1}^{t-1} b_t^*$), the manager will not bluff, because the auditor will work hard with probability one.

The owner benefits considerably from reputation formation by the auditor. Without an incentive to build a reputation, the weak auditor's optimal strategy is to shirk and collude with the manager at each stage t of the game since $\alpha_t > m_t$. Hiring such an auditor is of no economic benefit to the owner. The effect of reputation formation is to provide an incentive to the weak auditor to work hard and behave independently over a large number of periods in the game and as a result to reduce moral hazard on the auditor's act and report. This induces truthful reporting on the part of the manager and enables the owner to provide incentives for the manager to work hard. We had earlier shown that if investigation costs k are high it may not be possible for the owner to choose a θ_t sufficiently high that would force the auditor to work hard. In the presence of reputation effects, however, we observe that the auditor does work hard over several periods of the game. Thus the auditor's economic incentive to build a reputation enables the owner to reach a Pareto dominant equilibrium that cannot be reached if the only mechanism of enforcing contracts is via costly monitoring. Later we show that reputation formation can also substitute for costly monitoring.

3.4.5 REPUTATION BENEFITS WHEN WORKING HARD IS A DOMINANT STRATEGY FOR THE WEAK AUDITOR

In the game analysed in Propositions 3.4.1 and 3.4.2, we assumed that the penalty imposed on the ***weak*** auditor and the investigation cost, k, were such that $\alpha_t > m_t \; \forall \; t = 1,\ldots,T$. The question we address next is what happens to the nature of the equilibrium, if there exists a θ_t, investigation cost, k and penalty, g_t^2, such that $m_t > \alpha_t$ and $l_t > l_t + p_t^*(\phi_t - m_t)$, that is $m_t > \phi_t$ for all $t=1,\ldots.T$.

Assuming the owner chooses a θ_t so that $m_t > \alpha_t$ and $m_t > \phi_t$, the optimal strategy for the weak auditor is to work hard. Since there is no moral hazard problem for the weak auditor, it may appear that where such a θ_t exists, there is no role for reputation formation by the weak auditor. However, we show in Proposition 3.4.3 that choosing a θ_t so that $m_t > \alpha_t$ and $m_t > \phi_t$ may not be efficient. Where investigation costs k are high, the owner's optimal strategy is to choose a θ_t, with $\alpha_t > m_t$ and $\phi_t > m_t$, that provides incentives for the weak auditor to build a reputation. Reputation effects ensure that the weak auditor's optimal strategy is to work hard and the owner's expected investigation costs are lower. Reputation formation serves as a partial substitute for costly monitoring.

Not surprisingly, the equilibrium we obtain is very similar to the equilibrium of Propositions 3.4.1 and 3.4.2. We model the game exactly as before. We briefly give below the optimal strategies for the various players. The recursive definition of q_t is exactly as before.

STRATEGIES OF THE OWNER

(1) The owner contracts with the manager on reported outcomes. The owner writes a contingent contract as if the outcomes reported by the manager were the true outcomes. The contract the owner writes with the manager is a function of the history $(y_{pt}, \hat{x}_t, \hat{n}_t)$. (2) The owner pays the auditor in each period a fee g_t (g_t can of course vary from

period to period) and chooses an investigation strategy $\theta_t(y_{pt},\hat{x}_t,\hat{n}_t)$ which satisfies Condition (A). If the auditor shirks, the owner chooses $m_t > \alpha_t$ and $m_t > \phi_t$ for all other t.[17]

STRATEGIES OF THE AUDITOR

If a bad outcome is reported both types of auditors simply accept the manager's report. If a good outcome is reported and if the auditor is of the strong type, he always works hard as per the matrix of payoffs described earlier. If the auditor is of the weak type, the owner's strategies are such that the weak auditor either works hard or randomizes over all T periods of the game.

STRATEGIES OF THE MANAGER

The manager's strategy is exactly as described in Proposition 3.4.1 until the auditor shirks. Once the auditor shirks, the owner's strategy ensures that both the strong and weak auditor always works hard. Consequently, the manager's optimal strategy is to report truthfully and take the action desired by the owner.

PROPOSITION 3.4.3 The strategies and beliefs given above constitute a sequential equilibrium in the game among the owner, manager and auditor.

Proof: See Appendix 3.

If we further assume ***plausible beliefs*** (Kreps and Wilson, 1982a), we can show using the methods of Proposition 3.4.2 that the equilibrium described above is essentially unique. We close this section with a few comments on the model of Proposition 3.4.3.

In a game without reputation effects, the owner chooses θ_t so that in equilibrium $m_t > \alpha_t$ and $m_t > \phi_t$ $\forall$ $t = 1,\ldots,T$. This will ensure that working hard is a dominant strategy for the weak auditor at each stage t. The optimal strategy for the manager will then be to take the act desired by the principal and report truthfully. In a game with reputation effects such as the game of Proposition 3.4.3, the owner chooses the optimal investigation strategy θ_t to satisfy Condition (A) until the auditor shirks. He then chooses θ_t so that $m_t > \alpha_t$ and $m_t > \phi_t$ for all other t.

We can thus evaluate the benefit to the owner from reputation formation by the weak auditor. We see from Proposition 3.4.3 that in the game with reputation effects, $\forall$ t such that $\Pi_{i=1}^{t-1} b_i^1 \leq q_t$, the owner chooses a θ_t so as to ensure Condition A(i) is satisfied rather than a θ_t that ensures $m_t > \alpha_t$ and $m_t > \phi_t$. Condition A(i) is a weaker condition than $m_t > \alpha_t$ and $m_t > \phi_t$. Indeed, under condition A(i), α_t and ϕ_t may be greater than m_t. Consequently, the owner's optimal investigation strategy and investigation cost is smaller in the game where the auditor can build a reputation. The intuition is that the economic incentive for the auditor to build a reputation reduces the investigation cost that the owner must incur to monitor the auditor. Reputation formation serves as a partial substitute for costly monitoring. Condition A(i) suggests the benefit to the principal as a result of reputation building by auditors is $l_t - m_t$ for all time periods t for which $q_t \geq \Pi_{i=1}^{t-1} b_i^1$.

3.4.6 CLOSING COMMENTS ON BASIC MODEL

So far we have assumed no discounting on the part of the auditor. If the auditor discounts his payoffs by a factor ρ per period we have the following results.[18] If $\rho > (\phi_t - m_t)/[l_{t-1} + p^*_{t-1}(\phi_{t-1} - m_{t-1}) - \alpha_{t-1}]$ the equilibrium is precisely as described in Propositions 3.4.1 and 3.4.3, except that the randomizing probabilities are a little different. If $\rho \leq (\phi_t - m_t)/[(\phi_t - m_t + l_{t-1} + p^*_{t-1}(\phi_{t-1} - m_{t-1}) - \alpha_{t-1}]$, the equilibrium is very different. The weak auditor shirks in the very first period and the manager works hard and reports truthfully if $q_T > b^1_T$ and shirks and bluffs if $q_T < b^1_T$. This occurs because the low discount factor implies that the cost of reputation formation in early periods can not be recovered in subsequent periods. For ρ such that $(\phi_t - m_t)/[(\phi_t - m_t) + l_{t-1} + p^*_{t-1}(\phi_{t-1} - m_{t-1}) - \alpha_{t-1}]$ is less than $(\phi_t - m_t)/[(l_{t-1} + p^*_{t-1}(\phi_{t-1} - m_{t-1}) - \alpha_{t-1}]$, the manager will work hard and report truthfully for large t, but the equilibrium is more involved for small t. Thus for large discount factors ρ the equilibrium is very similar to the equilibrium without discounting.

For the reputation effect mentioned in the previous propositions to occur, it is crucial that it pay the weak auditor to emulate the strong one. The investigation cost, k, the penalty if auditor shirking is detected and the probability of detection, θ_t, play important roles in this scheme. The principal's conditional investigation decision (which in this simple model will only be invoked when a good outcome arises) ensures that the expected utility to the weak auditor from working hard exceeds his expected utility from shirking. The sequential equilibrium leads to the choice of an action that is not equilibrium behavior in the single stage game. The sequential equilibrium has the property that the manager works hard and reports truthfully and the auditor works hard and reports accurately over many stages of the game.

In the next section, we briefly state the nature of the equilibrium if hard-working auditors are not ***perfect***.

3.5 EXTENSIONS OF THE BASIC MODEL

In the equilibrium described in Section 4 we assumed "perfect" auditors so that if the auditor works hard, he is always able to determine the actual outcome. In this section, we continue to assume that the two types of auditors discussed in Section 3.4 exist, that is, the strong auditor who prefers to work and the weak auditor who prefers to shirk. In our first extension to the model of Section 3.4, we introduce a small probability γ that a hardworking auditor will be unable to determine the actual outcome (and possible bluffing by the manager). We further assume that if the auditor works hard, the owner's investigative act ζ reveals the auditor to have worked hard (and not shirked) even though the auditor is unable to detect the manager's bluff. In our second extension, we assume that the strong auditor is one who works with probability $1-\gamma$ and shirks with probability γ, that is, failure to pick up bluffing by the manager is viewed as shirking. This causes an interesting departure from our earlier analysis because shirking is no longer conclusive evidence of a weak auditor.

3.5.1 FIRST EXTENSION OF THE BASIC MODEL

We first consider the case where a hardworking auditor will, with probability γ, not detect bluffing by the manager. The manager's and auditor's expected utilities are identical to those described in Table 1 except for the manager's expected utility when the manager shirks and bluffs and the auditor works. If the manager shirks a good outcome occurs with probability p_t^1 and a bad outcome (which is reported by the manager as a good outcome) occurs with probability $1 - p_t^1$. If a bad outcome occurs which the manager reports as good, the auditor detects the bluff with probability $1-\gamma$ and fails to detect it with probability γ. The manager's expected utility is given by

$$p_T^1 b_t^{sg} + (1 - p_t^1)[(b_t^{sg} - \beta_t)(1 - \gamma) + \gamma b_t^{sg}]$$

$$= b_t^{sg} - \beta_t(1 - p_t^1)(1 - \gamma)$$

The equilibrium is exactly as described in Proposition 3.4.1 with b_t^1 replaced by $\frac{b_t^1}{(1-\gamma)}$ where $b_t^1 = (b_t^{sg} - b_t^{sb})/\beta_t$. Essentially, for all $q_t > \Pi_{i=1}^{t-1} \frac{b_i^1}{1-\gamma}$, the weak auditor and strong auditor always work hard. For $q_t \leq \Pi_{i=1}^{t-1} \frac{b_i^1}{1-\gamma}$, the weak auditor randomizes and the strong auditor works hard. For $q_t > \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$, the manager works hard and reports truthfully. For $q_t < \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$, the manager shirks and bluffs.

With the introduction of audit imperfections the cutoff point each period, above which the manager works hard and reports truthfully, increases. In other words, audit imperfections induce greater shirking and bluffing by managers. The reputation effect is somewhat reduced.

3.5.2 SECOND EXTENSION OF THE BASIC MODEL

In this case we assume the strong auditor is one who works with probability $1-\gamma$ and shirks with probability γ. This causes an interesting departure from our earlier analysis because shirking is no longer evidence of a weak auditor.

In the rest of this section we show that even with such an "imperfect" auditor, the equilibrium in the multi-period game is remarkably similar to the one discussed in Section 4. Unfortunately, the equilibrium to this game does not have the Markov property (with q_t being a sufficient statistic for the history of play up to period t) that characterized the equilibrium in the previous section. The action choices of the players depends upon the previous history of play up to period t. Giving a complete specification of the equilibrium that is obtained is extraordinarily tedious because it is based on some very involved recursions. So we give only a rough description of what happens.

We start by giving the function q_t. Set $q_T = \delta$. For $t < T$, we have the following recursive definition of q_t.

(i) If a bad outcome arises and the manager reports it as bad, the manager cannot update the probability of the auditor being strong, so $q_t = q_{t+1}$.

(ii) If the auditor works hard $q_t = \max\left(\Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}, q_{t+1}\right)$ where $b_i^1 < 1 - \gamma \;\; \forall\, i = 1,\ldots,T$.

(iii) If the auditor shirks, then q_t depends on the history of the game up to stage t + 1 (q_t also depends on the values of ϕ_i, α_i and m_i $\forall$ i = 1,...,t). Note that q_t never goes to 0 even if the manager detects auditor shirking. By our assumption, the strong auditor may shirk and bluff with probability γ.

When ϕ_i, i = 1,...,t is large relative to the other parameters, q_t is defined as follows.

If $q_{t+1} \geq \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$, then $q_t = q_{t+1}$.

If $q_{t+1} < \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$, then $q_t = \frac{\gamma q_{t+1}}{(1-q_{t+1})(1-r)+\gamma q_{t+1}}$

where $r = \frac{(1-\gamma)^{t+1} q_{t+1} - (1-\gamma)\ \Pi_{i=1}^{t} b_i^1 q_{t+1}}{(1-q_{t+1})\ \Pi_{i=1}^{t} b_i^1}$ = Prob(auditor works hard | auditor weak)

STRATEGIES OF THE OWNER. The owner's strategies are similar to those described in the previous section.

STRATEGIES OF THE AUDITOR

If a bad outcome is reported by the manager both types of auditors simply accept the report and $q_t = q_{t+1}$. If a good outcome is reported and (a) if the auditor is of the strong type, he works hard with probability $1-\gamma$ and shirks with probability γ.

(b) if the auditor is of the weak type, then if

(i) t = 1, the auditor shirks.

(ii) If $t > 1$, then, if $q_t < \Pi_{i=1}^{t-1} \frac{b_i^1}{1-\gamma}$, the weak auditor works hard with probability $\frac{q_t(1-\gamma)^t - \Pi_{i=1}^{t-1} b_i^1(1-\gamma)q_t}{\Pi_{i=1}^{t-1} b_i^1(1-q_t)}$ and shirks with complementary probability.

(iii) If $t > 1$ and $q_t \geq \Pi_{i=1}^{t-1} \frac{b_i^1}{1-\gamma}$, the weak auditor works hard with probability $1-\gamma$ and shirks with probability γ.

STRATEGIES OF THE MANAGER

(i) If $q_t > \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$, the manager always works and tells the truth.

(ii) If $q_t < \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$, the manager shirks and bluffs.

(iii) If $q_t = \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$, the manager randomizes.

<u>PROPOSITION 3.5.1</u>: The strategies and beliefs given above constitute a sequential equilibrium with the property that the manager works hard and reports truthfully over all periods of the game for which $q_t > \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$.

<u>Proof</u>: The proof parallels that of the earlier section, but is much more complicated because strategies at each stage depend on the history of the game up to that stage. Once again, if we assume ***plausible beliefs*** and if $\delta \neq \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$, for $t \leq T$, every sequential equilibrium has unique on-the-equilibrium path strategies.

The effect is that for sufficiently large periods of the game (that is, periods for which $\delta > \Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$), the probabilities associated with the auditor working hard are high enough for the manager not to shirk and bluff. This despite the fact that even the strong auditor shirks with probability γ.

In fact, it turns out that if q_t is sufficiently high (that is above a certain cut off point), the optimal strategy for the weak auditor is to shirk with probability one. The high probability that the auditor may be of the strong type ensures that the manager reports truthfully. Even if the manager observes the auditor shirking he will not assess q_{t-1} to be equal to zero since there exists a non-zero probability that the shirking auditor is the strong auditor. Thus, introducing γ allows us to introduce ideas about exploiting or using of reputation. In the game of Section 4, the weak auditor never shirks, however large be the value of q_t, since shirking behavior perfectly reveals the auditor's type.

3.6. COMPARATIVE STATICS

In this section we list some of the insights that the model provides in terms of understanding audit relationships.

(1) The model suggests that the value of reputation and the cost the auditor is willing to incur to achieve it increases with the frequency with which it may profitably be used. This is reflected in the desire of the weak auditor to maintain his reputation q_t early on in the game when there are many stages to go and to give up on some of it towards the end of the game. Being able to use reputation more frequently at the same point in time (via auditing many firms) or over

large periods of time, increases the incentives for building and maintaining a reputation.[19]

(2) For a given value of $q_T = \delta$, the longer the period T, the greater the periods over which the manager will not shirk (since δ will be greater than $\Pi_{i=1}^{t-1} b_i^1$ for a larger number of periods). Indeed in the limit as T goes to infinity, the manager will always work hard and report truthfully becasue δ will be greater than $\Pi_{i=1}^{t-1} b_i^1$ for all t.

If the auditor audited N firms each period for T periods, we can interpret the total number of periods in the game to be N*T (Wilson, 1983a). Larger N can then simply be interpreted as a larger number of plays of the game. As already indicated above, the larger the number of periods, the greater the incentives for the auditor to build and maintain a reputation. The advantage of building a reputation is directly related to the frequency with which it can be used. Having numerous clients enables an auditor to reinforce his reputation frequently (since the auditor will work hard in all periods t for which $\delta \geq \Pi_{i=1}^{t-1} b_i^1$). Higher frequency of audits enables the auditor to realize higher returns on his reputation per unit time and provides greater opportunities to enhance his reputation. Thus auditors with a large number of clients enjoy economies of specialization in building a reputation.

(3) For a fixed period T, the higher the value of $q_T = \delta$, the longer the periods over which the manager will not shirk and bluff.

(4) Lower values of b_i^1 tend to increase the value to the auditor of working hard (since working hard ensures that the manager will not shirk and bluff in this period and in the future). Further, for a given

probability of the auditor being a hard working type, low values of b_i^1 imply that the penalty from shirking is high, so that there is a greater disincentive for the manager to shirk and bluff. An alternative way to view this is that low values of b_i^1 results in δ being greater than $\Pi_{i=1}^{t-1} b_i^1$ over a larger number of periods. In each of these periods the auditor's optimal strategy is to work hard and report the outcome truthfully. Consequently, the manager's optimal strategy is also to work hard and report truthfully.

(5) If the weak auditor's disutility e_t - m_t from working hard rather than shirking decreases, the cost of building a reputation decreases. This provides the weak auditor with a greater incentive to build a reputation and increases the probability of the weak auditor working hard. Consequently, the probability of the manager shirking and bluffing decreases.

(6) In the case of the imperfect auditor, the smaller the value of γ, the lower the cutoff point each period for the manager to prefer to work and report truthfully rather than shirk and bluff. This strengthens reputation effects. Conversely, if auditors are not able to determine the correct outcome even after working hard (which correspond to large values of γ), managers will find it optimal to shirk and bluff even in early periods of the game. Large values of γ weaken reputation effects.

3.7 IMPLICATIONS FOR AUDITOR INDEPENDENCE

The objective of this section is to examine how our model relates to the issue of auditor independence. We shall use various definitions of independence proposed by Antle (1980) and interpret these definitions within the context of our model. Section 4.7.1 is devoted to formulating the definitions. In Section 4.7.2 we analyze these definitions.

3.7.1 DEFINITIONS OF AUDITOR INDEPENDENCE

This section is basically a review of Antle's definitions of auditor independence (see Antle [1980]). Antle classifies attempts to define auditor independence into two categories: (1) attempts to specify the relationships between (potentially) observable variables that determine whether an auditor is independent and (2) attempts which address the conceptual meaning of the term without particular attention to stating these concepts in terms of observables.

Defining Independence in Terms of Observables

This has been the focus of the AICPA and the SEC. These bodies seek to determine whether in specific circumstances the auditor is or is not independent. The operational definitions of the AICPA and SEC have centered on two issues:

1) financial interest in the client by the auditor, and

2) employment relationships of the auditor and his relatives

This led Antle to his first definition of auditor independence.

<u>Definition 1</u>: An auditor is independent (1) if the optimal incentive payments to the auditor g^1 and g^2 do not depend on the audit report $\hat{n}$.

Conceptual Definitions of Independence

While there is considerable ambiguity about the conceptual definitions of independence, the emphasis of various writers (see, for instance, Carey [1956], Burton [1974], and Mautz and Sharaf [1961]) appears to be on the performance of the verification work, objectivity, unbiasedness and honesty. This results in a second definition of independence that is a slight modification of Antle's second definition.

Definition 2: An auditor is independent (2) if there exists an optimal incentive plan that induces the auditor to work hard and report truthfully.

Our definition of independence (2) captures the auditor's investigative act as well. That is, we do not want an auditor to be classified as independent if he reports honestly what he knows by in fact "looking the other way". As Rappaport (1972) says

> The matter of independence tends to merge with the auditor's obligation to do a proper job. If an auditor has omitted a generally recognized procedure, has failed to ask perfectly obvious questions, has not insisted on disclosing important facts relating to the financial position or operating results of the enterprise being audited, it seems perfectly clear that such an auditor lacks independence even though he does not have financial interest in the enterprise, is not an officer, director or employee, and has no other relationship with the company which would preclude him from acting as its independent auditor (pp. 26,27).

The second conceptual definition of independence has to do with the ability of the auditor to withstand pressure by the manager to influence the audit or final report of the auditor to the owner (see for instance, Nichols and Price [1976, p.336] or Watts and Zimmerman [1976, p. 43]. Other authors define independence as "freedom from undue influence" (see for instance, Rittenberg [1977, p. 20].

In order to understand what constitutes "undue" influence within the context of our model, consider the case where the manager and auditor collude or select their strategies cooperatively. As Antle (1980) suggests the auditor and manager could form a coalition, and perhaps make side payments. The side payments could be viewed as the manager and auditor recontracting after the principal has structured their payoffs. Such recontracting may alter the auditor's incentive to work hard and report truthfully. This could be interpreted as the manager exercising undue influence on the auditor. We interpret Antle's third definition of independence very broadly in order to reflect these ideas.

Definition 3: An auditor is independent (3) if he and the manager play noncooperatively.

We proceed to analyze these definitions within the context of our model.

3.7.2 ANALYSIS OF AUDITOR INDEPENDENCE

The purpose of this section is to analyse the three definitions of auditor independence introduced in the previous section. We begin by examining independence (1) then turn to independence (2) and finally independence (3).

Independence (1)

The critical question underlying independence (1) is whether payments to the auditor should be made to depend on the audit report $\hat{n}$ in order to provide the auditor with incentives to work hard and report truthfully. It is important to emphasize that our only concern is with providing the auditor with incentives to work hard and report truthfully. We are not concerned with risk sharing between the owner and auditor. The latter is outside the purview of our model. The role of the auditor in our model is only to reduce or eliminate moral hazard on the manager's report and to facilitate thereby more efficient contractual arrangements between the owner and manager.

In Propositions 3.4.1 and 3.4.3 we demonstrate that paying the auditor a fixed fee g_t^1 each period that is not a function of $\hat{n}_t$, yields an equilibrium in which penalties and reputation effects ensure that the auditor does not shirk over a large number of periods in the game. Thus we are able to motivate the auditor to work hard and report truthfully without requiring g_t^1 to be a function of $\hat{n}_t$. Therefore, independence (1) is consistent with our model. If the auditor's contract did depend on $\hat{n}_t$, it could distort the auditor's incentives to report truthfully. We had seen in the case of the owner-manager model, that in the presence of moral hazard on communication, truthful reporting can best be induced by contracts which do not depend on $\hat{n}_t$.

Independence (2)

Independence (2) states that there exists an optimal incentive plan that induces the (weak) auditor to do the necessary investigative work and to report truthfully. Propositions 3.4.1 and 3.4.3. prove that, in equilibrium, reputation effects drive the auditor to work hard and report truthfully over many periods of the game.

It is important to emphasize the equilibrium nature of the argument. The weak auditor actually prefers to shirk. However, as the game is repeated over time the weak auditor finds it in his interest to work hard and report truthfully. That is, the auditor behaves as if he is independent (2) in our model over all but a few periods. Propositions 3.4.1 and 3.4.3 provide sufficient conditions for such behavior even though in fact the auditor prefers to shirk. This is the key idea. We do not postulate that a necessary characteristic of an auditor is that he work hard and report truthfully. Rather, we show that even auditors who prefer to shirk in fact work hard and report truthfully in equilibrium.

We close with one final comment about independence (2). Independence (2) does not explicitly allow for the possibility of collusion or cooperative play between the manager and auditor in the subgame. For instance, the auditor and manager could form a coalition and make side payments that may alter the auditor's incentive to work hard and report truthfully. We consider this case next in independence (3).

Independence (3)

Independence (3) states that the auditor and manager play non-cooperatively, that is, the manager does not exercise "undue" influence over the auditor. There are several situations in which cooperation or collusion between the auditor and manager could be detrimental to the owner. For example, the manager may offer side payments to the auditor to induce the auditor to alter his report. Alternatively, the auditor and manager could coordinate their strategies to escape penalties for shirking. We want our equilibrium to be immune to such cooperative behavior.

Indeed we show that multi-period reputation effects may preclude such behavior. Within the context of our model, agreeing to collude and not report a breach would reveal the auditor to be of the weak type (here defined to be one who is willing to collude). The optimal strategy for the manager thereafter would be to shirk, bluff and (attempt to) collude with the auditor at each stage of the game. Using arguments similar to those used in Proposition 3.4.1 and 3.4.3 it can be shown that, for sufficiently high q_t, the auditor's optimal strategy is to not collude but instead to report the breach (provided the auditor's expected utility from reputation formation and forcing the manager to report truthfully exceeds the auditor's expected utility from cooperating and colluding). The auditor's optimal strategy is to play non-cooperatively in each period and establish a reputation for independence (3) even though such play may be disadvantageous in the period. The owner anticipating cooperative play in the subgame between the manager and auditor chooses a game design in which he provides an incentive to the auditor to behave as if he is independent (3). In our model the benefit of building a reputation for non-cooperative play provides the auditor with incentives to guarantee his independence (3).

Sharing information and coordinating the auditor's and the manager's actions may improve the efficiency of the productive process. However, if the manager's contract depends on the auditor's report as in our model, some noncooperative behavior, as in independence (3), is desirable from the owner's point of view.

3.8 SUMMARY AND CONCLUSION

In this chapter we have demonstrated that the presence of information asymmetries leads the weak auditor to work hard over a number of periods even though such behavior is irrational in a single play of the game. The information asymmetry gives the manager reason to forecast future actions of the auditor on the basis of past behavior. This provides the weak auditor with an incentive to work hard, which in turn leads the manager to work hard and report truthfully over all but the last few periods.

Our motivation for audit firms of different types is that the audit firm is involved in a bigger game (involving audits of other clients both in this period and in future periods) of which the game studied here is only a part. The audit firm's strategy in the smaller game depends in part on the audit firm's strategy in the larger game. Thus, it is difficult for the manager to be certain that what is optimal for the auditor in the part-game (which is to shirk) is optimal in terms of the auditor's overall strategy. Consequently, the manager assesses some probability that the auditor will work hard and report truthfully.

Three factors in our model lead to the emergence of reputations. (1) Information asymmetry (namely that the manager is unsure about the possible motivations for the auditor's choice of strategies), (2) repeated actions, so that it becomes worthwhile to maintain or acquire a reputation, and (3) benefits to emulation of the strong auditor by the weak auditor. These conditions appear to be necessary and sufficient for reputation formation to occur in the auditing context. Since $\Pi_{i=1}^{t} b_i^1$ and $\Pi_{i=1}^{t} \frac{b_i^1}{1-\gamma}$ would asymptote to zero, it is clear that δ does not have to be very large for it to pay the weak auditor to emulate the strong auditor and build a reputation for hard work and independence.

We next examined the implications of our model for auditor independence. We used the three definitions of independence suggested by Antle [1980]. Independence (1) states that there are optimal auditor payments which are independent of the audit report. Our equilibrium is consistent with independence (1). Independence (2) states that there is an optimal incentive plan which induces the auditor to work hard and report truthfully. Reputation effects ensure that independence (2) is satisfied. Independence (3) states that the auditor and manager play non-cooperatively and do not collude in the subgame. Once again reputation effects provide the auditor with incentives that guarantee independence (3). Thus, within the context of our model, all three definitions of independence are satisfied in equilibrium.

Determining sequential equilibria is a complicated process. Consequently, we chose to confine ourselves to only two types of auditors and only two possible outcomes. However, we did retain the essential features of the structure of the problem such as moral hazard on both the manager's and auditor's actions and reports. Although defining the problem in this way did make our computations easier, we doubt that it is crucial to the intuition that the model provides.

It is important to stress that what drives the analysis is that the common knowledge assumption about the manager and auditor exactly knowing each other's payoffs, fails. The manager believes with probability δ that there exist incentives for the auditor always to work hard. This belief "forces" the manager to work and report truthfully. As we have shown, this is not an incredible or unbelievable threat. Even the weak auditor will work hard if called upon to do so. This is precisely why consideration of beliefs and strategies off the equilibrium path is important.

In the partial equilibrium model that we analyze, the owner benefits considerably from reputation formation by the auditor. We study the economic benefits to the owner in two settings. In Proposition 3.4.1 we consider the case where it is very costly for the owner to monitor the auditor's act and report. The weak auditor's optimal strategy in such a case is to shirk. However, we show that the future economic benefit of engaging in reputation formation may alter the weak auditor's behavior and provide him with an incentive to work hard. As a result the extent of moral hazard on the auditor's act is diminished. This enables the owner to reach a Pareto dominant equilibrium that can not be attained via costly mointoring alone. In Proposition 3.4.3 we consider a setting where moral hazard on the auditor can be eliminated via costly monitoring. We show that providing economic incentives for the auditor to build a reputation reduces the investigation costs that the owner must incur to monitor the auditor. In this case, reputation formation serves as a partial substitute for costly monitoring by the owner.

Our analysis suggests that there do exist advantages to not restricting either the number of audits, audit fees or other revenues that could be earned by auditors.[20] These provide important economic incentives to build and maintain a reputation. As suggested earlier, hiring an auditor who accurately reports the cash flow will enable the owner to write contingent contracts and provide incentives to the manager. This leads to Pareto improvement (the owner is made better off and the manager no worse off) if audit fees do not exceed the benefits of improved contracting.

TABLE 3.0

NOTATION FOR CHAPTER 3

p_t^1 = probability of a good outcome in period t if the manager shirks, i.e., chooses act a_t^1.

p_t^* = probability of a good outcome in period t if the manager works, i.e., chooses act a_t^*.

b_t^{wg} = the manager's expected utility in period t if the manager works hard and a good outcome results which the manager reports as good.

b_t^{wb} = the manager's expected utility in period t if the manager works hard and a bad outcome results which the manager reports as bad.

b_t^{sg} = the manager's expected utility in period t if the manager shirks and (i) a good outcome results which the manager reports as good or (ii) a bad outcome results which the manager reports as good without the misreport being detected.

b_t^{sb} = the manager's expected utility in period t if the manager shirks and a bad outcome results which the manager reports as bad.

$b_t^{sg} - \beta_t$ = the manager's expected utility in period t if the manager shirks, reports a bad outcome as good and is detected by the auditor.

m_t = the auditor's expected utility in period t if the auditor works hard.

e_t = the auditor's expected utility in period t if the auditor shirks and is not investigated.

ζ_t = information generated by the owner about the auditor's investigative act.

θ_t = probability with which ζ_t is purchased.

d_t = the auditor's expected utility in period t if the auditor shirks when the manager reports a bad outcome as good and ζ_t is purchased.

ε_t = the auditor's expected utility in period t if the auditor shirks when the manager reports correctly and ζ_t is purchased.

α_t = $\theta_t d_t + (1-\theta_t)e_t$.

l_t = $p_t^* m_t + (1-p_t^*)e_t$.

ϕ_t = $\theta_t \varepsilon_t + (1-\theta_t)e_t$.

TABLE 3.1

OWNER DOES NOT INVESTIGATE
AUDITOR'S STRATEGIES

MANAGER'S STRATEGIES	Works when Good reported Accepts when Bad reported	Shirks when Good reported Accepts when Bad reported
Manager shirks Reports Good when Good Reports Good when Bad	$p_t^1 b_t^{sg} + (1-p_t^1)(b_t^{sg}-\beta_t), m_t$	b_t^{sg}, e_t
Manager works Reports Good when Good Reports Bad when Bad	$p_t^* b_t^{wg} + (1-p_t^*) b_t^{wb}, p_t^* m_t + (1-p_t^*) e_t$	$p_t^* b_t^{wg} + (1-p_t^*) b_t^{wb}, e_t$

OWNER INVESTIGATES
AUDITOR'S STRATEGIES

MANAGER'S STRATEGIES	Works when Good reported Accepts when Bad reported	Shirks when Good reported Accepts when Bad reported
Manager shirks Reports Good when Good Reports Good when Bad	$p_t^1 b_t^{sg} + (1-p_t^1)(b_t^{sg}-\beta_t), m_t$	b_t^{sg}, d_t
Manager works Reports Good when Good Reports Bad when Bad	$p_t^* b_t^{wg} + (1-p_t^*) b_t^{wb}, p_t^* m_t + (1-p_t^*) e_t$	$p_t^* b_t^{wg} + (1-p_t^*) b_t^{wb}, \epsilon_t$

APPENDIX 3

Proof of Proposition 3.2.1

We use a backward induction approach by starting with the agent's last period effort choice and applying the sequential rationality requirement as the game moves backward in time. In the last period T, a sequentially rational strategy must satisfy constraint (ii) of (2.6) regardless of what happened earlier. Further, the sharing rule s_T, action a_T and report $\hat{x}_T$ must satisfy the first constraint of (2.6).

Let $[a_T(s_T), \hat{x}_T(s_T)]$ maximize the agent's expected last period utility when the last period sharing rule is s_T. If the agent's optimal $(a_T, \hat{x}_T)$ is unique for each s_T that the principal might offer, we simply solve for the last period's problem, that is, problem (2.6). If this set of optimal actions is not unique, we assume the agent will select that act which maximizes the principal's expected utility. Let $[a^*_T(s_T), \hat{x}^*_T(s_T)]$ be the optimal action in $[a_T(s_T), X_T(s_T)]$ that maximizes the principal's last period utility. Therefore, the optimal last period action depends only on the last period sharing rule that is offered.

We know from Antle (1980) Proposition 3.4, that for any s_T and $(a^*_T, \hat{x}^*_T(\bullet))$ which maximizes (2.6), there exists an s'_T s.t.

1. s'_T and (a^*_T, id^X_T) satisfies constraints 2.6(i) and 2.6(ii) and
2. s'_T and (a^*_T, id^X_T) achieves the same value of (2.6A) as s_T and $(a^*_T, \hat{x}^*_T(\bullet))$.

It is basically achieved by defining s'_T as $s'_T(x) = s_T(\hat{x}^*_T(x))\ \forall\ x$, so that (a^*_T, id^X_T) leads to the same payoff to the manager for every $x \in X$ under s'_T as $(a^*_T, \hat{x}_T(\bullet))$ does under s_T. Consequently, without loss of generality, we can focus on only those manager's contracts that are truth-inducing, that is, our problem in period T can be restated by substituting t = T in Problem (2.7).

$$\max_{s_t(id^X_t), a_t} \int G_t[x_t - s_t(id^X_T(\bullet))] f(x_t|a_t) dx_t \qquad (2.7(A))$$

subject to

$$\int W_t[s_t(id^X_t(\bullet))] f(x_t|a_t) dx_t - V_t(a_t) \geq O_t \qquad (2.7(i))$$

$$(a_t, id^X_t) \in \underset{A \times X}{\text{argmax}} \int W_t[s_t(id^X_t(\bullet))] f(x_t|a_t) dx_t - V_t(a_t) \qquad (2.7(ii))$$

Further, from Proposition (3.5) of Antle (1980) we conclude that the only feasible contracts in Problem (2.7) are those equal to a constant almost everywhere. The proof is fairly straightforward. A necessary condition for 2.7(ii) above, is that x_T maximizes $U_T(s_T(\hat{x}_T), a^*_T)$ for a.e. x_T. (2.8)

If s_T is not equal to a constant a.e. then there is a subset, say B of X, with positive measure s.t. there exists an $\hat{x}_T$ with $s_T(\hat{x}_T) > s_T(\hat{x}'_T)\ \forall\ \hat{X}'_T \in B$. Therefore, (2.8) will not be satisfied.

Suppose the argument is true with t periods to go. Therefore with t + 1 periods to go, maximizing expected utility over the t + 1 periods is the same as maximizing utility in the (t+1)st period since the expected utility over the last t periods does not depend on what happens in the (t+1)st period. Over the last t periods, the agent receives $\sum_{T-t+1}^{T} O_t$ so that he must receive O_{T-t} in the (t+1)st period, so that in the (t+1)st period the (sharing rule, action, reported cash flow) triplet must be a solution to problems (2.6) and (2.7). Once again, the only feasible contracts are those equal to a constant almost everywhere. Therefore, the argument is true for all t. Q.E.D.

Proof of Proposition 3.2.2: We once again use a sequentially rational argument and follow a backward induction approach.

In the last time period T a sequentially rational strategy must satisfy exactly the same equations as would be derived in a one period agency model corresponding to the last period utility functions of the principal and agent and the last period's production function, that is, we must satisfy equation (2.6(ii)) for the period T.

By using an identical argument as in Proposition 1, we see that the only feasible contracts in the last period are those equal to a constant almost everywhere.

Therefore, no matter what $y_{p,T-1}$ is observed by the principal over the last T-1 periods, the optimal contract in the last period is a constant payoff almost everywhere.

In period T-1, the optimal contract is once again a constant almost everywhere, and so on until period 1, independent of y_{pt} Q.E.D.

Proof of Proposition 3.4.1:

First we verify Bayesian consistency. If the manager reports a bad outcome as bad, nothing is learned about the auditor so $q_{t-1} = q_t$. If a good outcome is reported and if $q_t \geq \Pi_{i=1}^{t-1} b_i^1$, where $b_i^1 = (b_i^{sg} - b_i^{sb})/\beta_i$ both the strong and weak auditor always work hard, so naturally, we learn nothing about auditor type. For $q_t \in (0, \Pi_{i=1}^{t-1} b_i^1)$ we have some probability that the auditor shirks, and some that he works hard. Once shirking takes place, we know for sure that the auditor is weak since the strong auditor would, given his payoffs, always work hard. If the auditor works hard, Bayes' rule gives us

$$q_{t-1} = \text{Prob(auditor strong} \mid \text{auditor works hard)}$$

$$= \frac{1.q_t}{1.q_t + [(\Pi_{i=1}^{t-1} b_i^1)q_t/(1 - q_t)\Pi_{i=1}^{t-1} b_i^1](1 - q_t)} = \prod_{i=1}^{t-1} b_i^1$$

Behavior of the auditor off the equilibrium path, viz $q_t \geq \Pi_{i=1}^{t-1} b_i^1$ and the auditor shirking, and $q_t = 0$ and the auditor working hard, result in $q_{t-1} = 0$.

The owner is content to play the strategies described earlier since the utility to him of employing the auditor and creating incentives for the manager to take the more productive act, is greater than the utility to the owner from paying the manager a fixed amount and not employing the auditor.

For the manager, if $q_t > \Pi_{i=1}^{t} b_i^1$, tedious computations show that the expected payoffs to the manager from shirking and bluffing (or working and bluffing) are less than the payoffs from taking the action desired by the owner and reporting the correct cash flow. (Recall that for $q_t \geq \Pi_{i=1}^{t-1} b_i^1$, the weak auditor and strong auditor always work hard, and between $\Pi_{i=1}^{t} b_i^1$ and $\Pi_{i=1}^{t-1} b_i^1$, the probability of the weak auditor shirking is not high enough to justify a shirk and a bluff from the manager. At $q_t = \Pi_{i=1}^{t} b_i^1$, the manager is indifferent to taking the action desired by the owner, or shirking and bluffing.

If $q_t < \Pi_{i=1}^{t} b_i^1$, he definitely finds it worthwhile to shirk and bluff (i.e., report a good outcome if a bad outcome results).

The strong auditor is playing optimally because at each stage game working hard is his optimal strategy. The expected utility to the weak auditor from stages t to 1 by following the strategy above is given by the following function va

$$va_t(q_t) = \sum_{i=\tau(q_t)}^{t} l_i + \phi_{\tau(q_t)} - m_{\tau(q_t)} + \sum_{j=1}^{\tau(q_t)-1} \alpha_j \qquad \text{if } \Pi_{i=1}^{t} b_i^1 < q_t = \Pi_{i=1}^{\tau(q_t)-1} b_i^1$$

$$= \sum_{i=\tau(q_t)}^{t} l_i + p^*_{\tau(q_t)}\left(\phi_{\tau(q_t)} - m_{\tau(q_t)}\right) + \sum_{j=1}^{\tau(q_t)-1} \alpha_j \qquad \text{if } \Pi_{i=1}^{t} b_i^1 < q_t < \Pi_{i=1}^{\tau(q_t)-1} b_i^1$$

$$= \phi_{t+1} - m_{t+1} + \sum_{j=1}^{t} \alpha_j \qquad \text{if } \Pi_{i=1}^{t} b_i^1 = q_t \text{ and the manager reports truthfully in period } (t+1).$$

$$= \alpha_{t+1} - m_{t+1} + \sum_{j=1}^{t} \alpha_j, \qquad \text{if } \pi_{i=1}^{t} b_i^1 = q_t \text{ and the manager bluffs in period } (t+1)$$

$$= \sum_{j=1}^{t} \alpha_j \qquad \text{if } q_t < \Pi_{i=1}^{t} b_i^1$$

We next show that given the function va described above, the weak auditor's strategies described earlier are indeed equilibrium strategies.

(i) If $q_t = 0$ and the weak auditor works hard, his utility is m_t this period (assuming $m_t < \alpha_t$) and $\Sigma_{j=1}^{t-1}\alpha_j$ in future periods. Consequently, the weak auditor prefers to shirk and the manager shirks and bluffs.

(ii) If $q_t > \Pi_{i=1}^{t-1} b_i^1$ and the weak auditor works hard, the manager will work hard and report truthfully. The weak auditor's utility is l_j each period for the rest of the game for as long as $q_j > \Pi_{i=1}^{j-1} b_i^1$ and some amount $\leq \phi_j$ when $q_j \leq \Pi_{i=1}^{j-1} b_i^1$. With such payoffs, the weak auditor prefers to work.

(iii) If $0 < q_t < \Pi_{i=1}^{t-1} b_i^1$, the weak auditor follows the randomizations discussed earlier, that make him indifferent between working hard or shirking. The manager works hard and reports truthfully for all j such that $q_j > \Pi_{i=1}^{j} b_i^1$, shirks and bluffs if $q_j < \Pi_{i=1}^{j} b_i^1$ and randomizes if $q_j = \Pi_{i=1}^{j} b_i^1$.

We therefore conclude that the weak auditor's strategies described earlier are equilibrium strategies. Q.E.D.

<u>Proof of Proposition 3.4.2:</u>

(i) It follows from Proposition 3.4.1 that the owner's optimal strategy is to provide incentives to the manager to work by satisfying condition (A).

(ii) It is clear from the payoff matrix that the strategy for the strong auditor is to always work.

(iii) We next show that the expected utility function $va_t(q_t)$ of the weak auditor is non-decreasing in q_t.

We prove this statement by induction using the valuation function $va_t(q_t)$. The statement is clearly true for t=1 (which is the last period). For $q_1 > b_1^1$, the weak auditor's expected utility is $l_1+p_1^*(\phi_1-m_1)$ and for $q_1 < b_1^1$, the weak auditor's expected utility is α_1, with $\alpha_1<l_1+p_1^*(\phi_1-m_1)$. For $q_1 = b_1^1$ the manager randomizes and the weak auditor's expected utility is between α_1 and $l_1+p_1^*(\phi_1-m_1)$. We assume the claim is true for t and prove the claim for t+1.

In equilibrium, for all $q_{t+1} > \pi_{i=1}^t b_i^1$, the weak auditor always works hard when a good outcome is reported, and $q_{t+1} = q_t$. Therefore, in period t+1, the weak auditor's expected utility increases by l_{t+1} for all $q_{t+1} > \pi_{i=1}^t b_i^1$. Since by the induction hypothesis, the weak auditor's payoff $va_t(q_t)$ is non-decreasing in q_t, it follows that for $q_{t+1} > \pi_{i=1}^t b_i^1$, $va_{t+1}(q_{t+1})$ is non-decreasing in q_{t+1}.

For $q_{t+1} < \pi_{i=1}^t b_i^1$, the weak auditor randomizes. If the weak auditor works hard $q_t = \pi_{i=1}^t b_i^1$ for all $q_{t+1} < \pi_{i=1}^t b_i^1$. In stage t+1, if $\pi_{i=1}^t b_i^1 > q_{t+1} \geq \pi_{i=1}^{t+1} b_i^1$, the manager works hard and tells the truth, so that the weak auditor's expected utility increases by l_{t+1}. In period t, $q_t = \pi_{i=1}^t b_i^1$, if the manager reports a good outcome and the auditor works, and $q_t = q_{t+1}$ if the manager reports a bad outcome which the auditor accepts. In either case $q_t \leq \pi_{i=1}^t b_i^1$. When $q_{t+1} > \pi_{i=1}^t b_i^1$, $q_t > \pi_{i=1}^t b_i^1$. Since $va_t(q_t)$ is non-decreasing in q_t, it implies that for $q_{t+1} > \pi_{i=1}^{t+1} b_i^1$, $va_{t+1}(q_{t+1})$ is non-decreasing in q_{t+1}. Finally, if $q_{t+1} < \pi_{i=1}^{t+1} b_i^1$, the weak auditor's utility is

$\alpha_{t+1} < l_{t+1}$, and in period t, $q_t \leq \pi_{i=1}^{t} b_i^1$. We therefore conclude that $va_{t+1}(q_{t+1})$ is non-decreasing in q_{t+1}.

(iv) Finally, if the manager shirks and bluffs at stage t (with $q_t < \Pi_{i=1}^{t} b_i^1$), and the auditor works hard and detects the bluff, then at stage t - 1, the manager must tell the truth with probability $(\alpha_t - m_t)/[l_{t-1} + p_{t-1}^{*}(\phi_{t-1} - m_{t-1}) - \alpha_{t-1}]$. If the manager reports truthfully at stage t (with $\Pi_{i=1}^{t} b_i^1 < q_t < \Pi_{i=1}^{t-1} b_i^1$), and the auditor works hard, then at stage t-1, the manager must tell the truth with probability $(\phi_t - m_t)/[l_{t-1} + p_{t-1}^{*}(\phi_{t-1} - m_{t-1}) - \alpha_{t-1}]$.

The proof of the above statement is relatively straightforward. If $q_t < \Pi_{i=1}^{t-1} b_i^1$ and the auditor works hard, the manager assesses q_{t-1} to equal $\Pi_{i=1}^{t-1} b_i^1$ in period t-1. This follows by applying Bayes' rule and checking for incentive compatibility. In other words, the weak auditor works hard in period t if and only if the 'reputation' he builds as a result forces the manager to randomize so that the manager works hard and reports truthfully with some positive probability in period t-1. The probabilities given in (iii) above, ensure that the loss in payoff to the auditor from working hard in stage t are exactly recouped in the next stage, hence the uniqueness of the equilibrium strategies. Q.E.D.

<u>Proof of Proposition 3.4.3</u>

The proof parallels that of Proposition 3.4.1. Baynesian consistency can be verified along the lines of Proposition 3.4.1.

It is clear from the description of the strategies for the owner, manager and auditor, that the equilibrium of Proposition 3.4.3 is a sequentially rational equilibrium. The owner is content to play the strategies described earlier, since the expected utility to him of employing the auditor and creating incentives for the manager to take the more productive act is greater than the expected utility to the owner from paying the manager a fixed amount and not employing the auditor.

The strong auditor is playing optimally, because at each stage game working hard is his optimal strategy, and by shirking at any stage he could be revealed to be the weak type.

The weak auditor's expected utility from following the strategy above results in a higher payoff at each stage game and the continuation game when compared to his expected utility from deviating at any stage. Since the optimal strategy of the weak and strong auditor is to work hard, the optimal strategy for the manager is to take the act a_t^* desired by the owner and report the outcome truthfully. Thus the strategies described are equilibrium strategies. Q.E.D.

ENDNOTES TO CHAPTER 3

1 Strictly speaking, x_t would not be observable by the manager. The manager's action would induce another random variable $m_t = m_t(a_t, w_t)$ which could be interpreted as earnings under generally accepted accounting principles or as statistically related to x_t. The manager would then report $\hat{m}_t(m_t)$ on which the payment $\hat{s}_t$ would be made. Yet another interpretation could be that the owner only receives a dividend payment based on the reported earnings, say $\bar{d}_t(x_t)$ rather than $(x_t(\cdot) - s_t(\cdot))$. Alternatively, we could visualize the manager inflating cash flows, for instance, by reporting sale of assets as operating cash flows on which the share s_t is calculated. We abuse notation and write $s_t(\hat{m}_t(m_t(\cdot)))$ as $\hat{s}_t(\hat{x}_t(\cdot)))$. This would not change any of the subsequent analysis.

2 Ng and Stoechenius (1979) further show that even if restrictions such as adequate cash flow to cover the manager's remuneration and unbiasedness of the report are imposed, the basic result of Proposition 1 would still hold in a single period model. Assuming sequential rationality, this result will carry forward to a multi-period setting.

3 Another alternative could be that instead of hiring an auditor to produce truthful reports, the owner could write a contingent contract on reported outcomes and threaten to sue the manager to keep him from bluffing. Typically however, the manager's wealth is small (unlike the "deep pockets" of auditors), so that with costly investigation the costs to the owner from a suit could easily outweigh the benefits. Consequently the threat would not be credible. Furthermore, in this game, managers have no incentives to form reputations and no other contracts other than those of Proposition 1 will be offered.

4 If the auditor shirks and does not observe the cash flow he simply reports the cash flow reported by the manager.

5 More generally ζ can be a function of $c, x, w, \hat{n}$ and $\hat{x}$. We do not write it as such to save notation.

6 We do not allow the manager's payment to be conditional on ζ. If we did it would not change the nature of the subsequent analysis but considerably complicate the notation. Our simplification may, however, be justified on two other grounds. One justification is that litigation against the auditor typically occurs after the manager has left the firm. The second argument is that if the manager's wealth is small (unlike the 'deep pockets' of auditors) the cost of litigation against the manager may outweigh the benefits.

7 Since the auditor does not observe the cash flow perfectly, he may not observe some elements of X_{t-1}.

8 If $b_t^{sg} - \beta_t > b_t^{sb}$, $p_t^1 b_t^{sg} + (1-p_t^1)(b_t^{sg} - \beta_t) > p_t^1 b_t^{sg} + (1-p_t^1) b_t^{sb} = p_t^* b_t^{wg} + (1-p_t^*) b_t^{wb}$, that is the manager's utility from shirking and bluffing exceeds the manager's utility from working and reporting truthfully.

9 We later determine the optimal θ_t.

10 This will occur for instance, if the cost of investigation, k, is very large in relation to the expected benefit to the owner from ζ_t. The expected benefit accrues from (a) providing incentives that motivate the manager to work hard and report truthfully and (b) penalties payable by the auditor to the owner. Penalties are likely to be lower if, as we assume, ζ_t is not a perfect indicator of auditor shriking.

11 Two people 1 and 2 are said to have common knowledge of an event E if both know it, 1 knows that 2 knows it, 2 knows that 1 knows it, 1 knows that 2 knows that 1 knows it, and so on.

12 Some multi-period studies of auditing have not allowed for any monopoly rents or returns to reputation (DeAngelo 1981a, 1981b) arguing that the expected returns to auditors from future audits is zero. This arises because of the competitive market structure assumed.

13 It is clear from the payoff matrix, that if the auditor is content to play his dominant strategy, the manager has no incentive to build or maintain a reputation. The payoff b_t^{sg}, when both the manager and auditor shirk is the highest payoff the manager can get. It is the auditor who would first like to invest in a reputation. An interesting problem is to examine the incentives for the manager to invest in reputation formation after the auditor has started building a reputation of his own. We capture this to some extent by assuming that the manager's compensation in any period is a function of the entire history of outcomes up to that period. However, we do not allow for different types of managers.

14 If a θ_i does not exist that satisfies this inequality, the weak auditor will always prefer to shirk. Since $\alpha_t > m_t \; \forall \; t$, the manager will shirk and bluff and there will be no value to hiring the auditor. A θ_i may not exist if the cost of investigation k is very large relative to the expected benefit to the owner from obtaining the report.

15 These conditions can be weakened. Condition (i) need not hold for all $t \geq \tau(q)$. Condition (ii) is also not necessary. However, without these conditions the equilibrium is considerably complicated.

16 This condition can be weakened as in the previous note.

17 These conditions can be somewhat weakened as discussed in Proposition 3.4.1.

18 In our discussion we assume that the manager has worked hard and reported truthfully in the previous period.

19 This is perhaps one reason why managers may have less of an incentive to build or maintain reputations. The length of their horizon and the frequency with which their reputation can be used appears to be limited. We propose to explore this issue later.

20 On the other hand unrestricted growth in auditor size may result in monopoly pricing inefficiencies.

CHAPTER 4

ENTREPRENEUR - AUDITOR - INVESTORS MODEL WITH ADVERSE SELECTION

In Chapter 3, we considered a multi-period owner-manager-auditor model in which the owner moves first to design incentive contracts for the manager and auditor who then take actions and issue reports under moral hazard. In this chapter we consider a setting in which an owner, who we henceforth refer to as the entrepreneur, does not have a first opportunity to put an incentive scheme into operation. The context is one in which the entrepreneur has already invested in a project and wishes to issue shares to outside investors. An adverse selection problem arises in that the entrepreneur has superior information about the project's quality.[1]

Leland and Pyle (1977) examine such a setting and show that the fraction of shares retained by the entrepreneur can signal the project's quality. However the entrepreneur is forced to retain more shares than he otherwise would in order that the project not be undervalued by investors. This leads to a risk sharing related loss in efficiency for the entrepreneur relative to the case where project quality can be costlessly communicated to investors. We expand this setting so the entrepreneur also has the option of hiring an auditor to verify and report to investors about the project's quality.

For expositional clarity we consider only a single period of the multiple periods in which the auditor is hired. Consequently, we do not explicitly model the effects on auditor behavior of reputation and penalty considerations as we did in Chapter 3. Instead, we assume sufficient incentives exist of the type described in Chapter 3, outside of the model, for the auditor to work hard and report truthfully in the period under consideration.

In the next section we review the entrepreneur-investor model of Leland and Pyle (1977). In Section 4.3 we describe and analyze the game among the entrepreneur, auditor and investors. In Section 4.4 we introduce a second auditor and examine the entrepreneur's strategic decision about the choice of auditor. Comparative statics results are derived in Section 4.5.

4.2. ENTREPRENEUR-INVESTORS MODEL

We will cast the Leland and Pyle model as an extensive form game keeping the spirit of Leland and Pyle's analysis. Consider a game among an entrepreneur and investors. The entrepreneur wishes to undertake an investment project. To finance the expenditure the entrepreneur contemplates selling a portion of the outcome claim to potential investors, J. The project requires a capital outlay, $\bar{K}$ and generates a gross return equal to $\mu + \tilde{v}$, where μ is the expected end of period value of the project and $\tilde{v}$ is a random variable with zero mean and strictly positive variance σ_v^2. $\tilde{v}$ represents the randomness in the project's returns.

We assume (1) the entrepreneur exhibits constant absolute risk aversion with a coefficient of risk aversion $a > 0$, (2) returns on the project and market are normally distributed and (3) the project is valued by investors based on the Capital Asset Pricing Model (CAPM).

We next describe the sequence of choices in the game. At the start of the game the entrepreneur obtains information about $\mu \in M^A \subseteq R$. There are a large number of poor quality projects in M^A. Investors know only the cumulative distribution of μ, denoted by $F(\mu)$. We shall sometimes refer to μ as characterizing the 'type' of the entrepreneur. As in Leland and Pyle $F(\mu)$ is a twice differentiable concave function. The number of projects with a value μ or more falls as μ increases.

Denote the random return on the market portfolio by $\tilde{M}$. The mean of $\tilde{v}$, the variance σ_v^2, and the covariance of $\tilde{v}$ with $\tilde{M}$ are common knowledge. Investors use ψ, the fraction of equity in the project retained by the entrepreneur[2] to determine the price $\underline{p}_j$ to offer for the project. After observing μ and the price schedule $\underline{p}_j(\psi)$ of each investor j, the entrepreneur chooses ψ and ν, the fraction of the market portfolio held by the entrepreneur. For ease of exposition and without loss of generality, we shall restrict ourselves to only two investors (J=2).

A project of type μ is valued equally by each potential investor j=1,2. The valuation VA, measured in dollars is an increasing function of μ and (possibly) also of the intensity of the signal ψ. Assuming competitive capital markets it can be shown via arbitrage arguments that $VA(\mu;\psi)$ is given by

$$VA(\mu;\psi) = \frac{1}{(1+r)} [\mu(\psi) - \lambda] \qquad (4.1)$$

where r denotes the riskless interest rate, $\mu(\psi)$ is a valuation schedule expressing the investors' perception of the expected end of period value as a function of ψ and λ is the market's adjustment of the risk of the project with returns $\tilde{v}$ about the mean. In the case of the Capital Asset Pricing Model, which is what we assume, $\lambda = \lambda^* \operatorname{cov}(\tilde{v},\tilde{M})$ where λ^* is the market price of risk.

4.2.1 STRATEGIES AND PAYOFFS OF THE ENTREPRENEUR AND INVESTORS

Investor j chooses the price schedule $\underline{p}_j$ to offer for the shares as a function of ψ, $\underline{p}_j(\psi)$. Investors engage in Bertrand-style competition between themselves.

DEFINITION: A pure strategy for investor $j=1,2$ is the choice of a measurable function $\underline{p}_j(\cdot)$ mapping each ψ that the investor may observe into a price offer $\underline{p}_j$, that is, $\sigma_j : C^A \to \underline{p}_j(\cdot)$, where $\underline{p}_j(\cdot) : [0,1] \to P_j^A$, the set of all prices $\underline{p}_j$ that investor j may offer. C^A is the investor's information set when he chooses $\underline{p}_j(\cdot)$. C^A includes everything that is common knowledge: $\bar{K}$, r, λ, σ_v^2, σ_M^2, $\operatorname{cov}(\tilde{v},\tilde{M})$, and the parameters of the entrepreneur's utility function. The set of pure strategies for investor j, $j=1,2$, is denoted by Σ_j.

<u>DEFINITION</u>: A pure strategy σ_E for the <u>entrepreneur</u> is a measurable function mapping each μ given $\underline{p}_1(\psi)$ and $\underline{p}_2(\psi)$ into a choice of ψ and ν, that is, $\sigma_E : M^A \times P_1^A \times P_2^A \to [0,1] \times [0,1]$. The set of pure strategies for the entrepreneur is denoted Σ_E.

The entrepreneur chooses ψ and sells his shares to whichever investor offers the highest price. We denote this price by $\underline{p}$. In case of a tie between the price offers of the two investors, the entrepreneur sells his shares to any or all of them in some random manner.

The entrepreneur's payoff will depend on μ, ψ and ν. The entrepreneur is presumed to maximize the expected utility of the end of period outcome $\tilde{W}_1$ with respect to his holding of equity in the firm and the market portfolio. His choices must satisfy his budget constraint

$$\tilde{W}_0 + (1-\psi)\,\underline{p} - \nu V_M - Y - \tilde{K} = 0. \tag{4.2}$$

if he undertakes the project and sells shares to outside investors.

$\tilde{W}_0$ is the money available to the owner-manager before investing in the project.

V_M is the market value of the market portfolio.

Y is the cost of his holdings of the riskless asset.

$\tilde{K}$ is the capital outlay for the project.

The outcome $\tilde{W}_1$ is determined by the entrepreneur's returns from investments in the firm, market and riskless security:

$$\tilde{W}_1 = \psi(\mu+\tilde{v}) + \nu\,\tilde{M} + (1+r)\,Y.$$

Substituting for Y from (4.2) yields

$$\tilde{W}_1 = \psi[\mu+\tilde{v}-(1+r)\underline{p}] + \nu[\tilde{M} - (1+r)V_M] + W_0(1+r) + (1+r)\underline{p} \qquad (4.3)$$

where $W_0 = \bar{W}_0 - \bar{K}$.

Under our assumption of constant absolute risk aversion, the entrepreneur's expected utility is given by

$$E[U(\tilde{W}_1)] = G[E(\tilde{W}_1) - \frac{a}{2}\,\sigma^2(\tilde{W}_1)]$$

where a is the coefficient of absolute risk aversion.

Therefore,

$$E[U(\tilde{W}_1;\mu,\sigma_E,\sigma_1,\sigma_2)] = G\{\psi[\mu - (1+r)\underline{p}] + \nu[\bar{M} - (1+r)V_M] + (1+r)\,W_0$$

$$+ (1+r)\underline{p} - \frac{a}{2}\,[\psi^2\sigma_v^2+\nu^2\sigma_M^2+2\psi\nu\,\mathrm{cov}(\tilde{v},\tilde{M})]\} \quad (4.4)$$

where G is a monotonically increasing function, σ_v^2 is the variance of v and σ_M^2 is the variance of the market portfolio.

Investor j's payoff j=1,2 and $i \neq j$ is given by[3]

$$U_j(\sigma_j,\sigma_E,\sigma_i) = \begin{cases} (1-\psi)\,[VA(\mu;\psi) - \underline{p}_j(\psi)], & \text{if he gets shares in the firm} \\ 0 & \text{otherwise} \end{cases}$$

4.2.2 NASH EQUILIBRIUM OF ENTREPRENEUR-INVESTORS GAME

DEFINITION: $(\sigma_E^*, \sigma_1^*, \sigma_2^*)$ is a pure strategy Nash equilibrium if

(i) $E[U(\tilde{W}_1; \mu, \sigma_E^*, \sigma_1^*, \sigma_2^*] \geq E[U(\tilde{W}_1; \mu, \sigma_E, \sigma_1^*, \sigma_2^*)]$

for all $\sigma_E \in \Sigma_E$ and for all $\mu \in M^A$.

(ii) For $i, j = 1, 2$ and $i \neq j$

$U_j(\sigma_j^*, \sigma_E^*, \sigma_i^*) \geq U_j(\sigma_j, \sigma_E^*, \sigma_i^*)$

for all $\sigma_j \in \Sigma_j$

Condition (i) requires that given investors' strategies, the strategy σ_E^* is optimal. The strategy σ_E^* defines the entrepreneur's strategy for all $\mu \in M^A$ that the entrepreneur may observe. Condition (ii) requires that for $j=1,2$, investor j's strategy σ_j^* is optimal given the strategies of the entrepreneur and the other investor. The strategy σ_j^* defines the price function $\underline{p}_j(\cdot)$ for all $\psi \in [0,1]$.

DEFINITION: We define a price function $\underline{p}(\psi)$ as informationally consistent if

(i) The entrepreneur chooses ψ to maximize $E[U(\tilde{W}_1)|\psi, \underline{p}(\psi)]$, that is, with prices conditioned on ψ the entrepreneur chooses a utility maximizing value of ψ, and

(ii) $\underline{p}(\psi) = VA(\mu;\psi)$, that is, investors purchase shares from the entrepreneur with the belief that the price $\underline{p}(\psi)$ reflects the value of the project. In other words, the μ that is inferred by investors based on ψ is in fact the actual μ of the project and the eqilibrium price $\underline{p}(\psi)$ equals the project's value.

In Proposition 4.2.1 we describe a Nash equilibrium for the game. The equilibrium is also an informationally consistent equilibrium.

PROPOSITION 4.2.1

The following strategies of the entrepreneur and each investor j constitute an informationally consistent Nash equilibrium in the game.

Investor j's Strategy

For $\psi > 0$, each investor j offers a price function

$$\underline{p}_j(\psi) = VA(\mu;\psi) = \frac{1}{(1+r)}[\mu(\psi) - \lambda]$$

where $\mu(\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + (1+r)\bar{K} + \lambda$ (4.5)

and $\bar{Z} = [\sigma_v^2\sigma_M^2 - \text{cov}(\tilde{v},\tilde{M})^2]/\sigma_M^2$.

For $\psi = 0$, $\underline{p}_j(\psi) = 0$

<u>Entrepreneur's Strategy</u>

After observing μ, $\underline{p}_1(\cdot)$, $\underline{p}_2(\cdot)$, the entrepreneur chooses ψ (and ν) using equation (4.5),

$$\mu = \mu(\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + (1+r)\bar{K} + \lambda$$

provided $\mu > (1+r)\bar{K} + \lambda$.
If $\mu \leq (1+r)\bar{K} + \lambda$, the entrepreneur does not undertake the project.

<u>Proof</u>: See Appendix 4

Proposition 4.2.1 demonstrates the existence of a Nash equilibrium that is informationally consistent. Since $\mu(\psi)$ is a strictly increasing function, the equilibrium has the property of perfectly signaling μ via ψ. In other words, the equilibrium is a fully separating equilibrium. Investors read a larger ownership fraction as a signal of higher project quality. The entrepreneur, in turn, is motivated to choose a larger fraction of ownership for a project with a larger μ.

Leland and Pyle further argue (p. 379) that the equilibrium of Proposition 4.2.1 is unique in the class of informationally consistent equilibria with price $\underline{p}(\psi)$ conditional on ψ. Roughly their argument is as follows. Any informationally consistent equilibrium conditioned on ψ must satisfy the differential equation $(1-\psi)\mu_\psi = a\psi\bar{Z}$. See also Riley (1979, p. 340). Therefore, $\mu = \mu(\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + C_0$. If C_0 is greater than $(1+r)\bar{K} + \lambda$, the equilibrium is obviously not informationally consistent. This follows because choosing $\psi = 0$ for projects with $VA(\mu) < \bar{K}$ yields $\underline{p}(0) > VA(\mu;0)$. If C_0 (equal to $\bar{C}_0$) less than $(1+r)\bar{K} + \lambda$ is chosen by investor i, the second investor j can offer a $\underline{p}_j(\psi)$ schedule that makes both the entrepreneur and second investor better off. See Leland and Pyle (1977, p. 379). Therefore the only informationally consistent equilibrium has

$$\mu(\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + (1+r)\bar{K} + \lambda. \qquad (4.5)$$

We close this subsection with several comments on the equilibrium of Proposition 4.2.1.

(1) If $\psi = \psi^*(\mu)$ is the level of signal chosen by entrepreneur of type μ according to equation (4.5), and $\underline{p}(\psi)$ is informationally consistent (as in Proposition 4.2.1), the expected utility of the entrepreneur of type μ is strictly lower for all $\psi \neq \psi^*(\mu)$. This follows immediately from our method of proof in Proposition 4.2.1. The entrepreneur of type μ chooses ψ to maximize his expected utility given that $\underline{p}(\psi)$ is informationally consistent. See also Riley (1979, p. 340).

(2) In equilibrium with signaling via ψ, the entrepreneur makes a larger investment in his own project than he would if the project value could be costlessly communicated to investors. See Leland and Pyle (1976, p. 376). This means that signaling is strictly second best in an ex-ante classically efficient sense as defined in

Holmstrom and Myerson (1981)[4]. Signaling μ to the market through ψ leads to a loss in efficiency since the entrepreneur is forced to invest in his own firm beyond that which is optimal if μ can be costlessly communicated to investors without any incentive problems. This is the familiar result of signaling models; excessive use of the signal in order to distinguish oneself from less preferred possibilities (in this case, smaller values of μ).

(3) It follows from equation (4.5) that $VA(0) = \bar{K}$ and $VA(\psi) > \bar{K}$ for $\psi > 0$. An implication of this is that a project will be undertaken if, and only if, its valuation by investors given μ exceeds its cost $\bar{K}$. Only projects with $\psi > 0$ which correspond to projects with a valuation $VA > \bar{K}$ are undertaken. Projects with a valuation $VA \leq \bar{K}$ are not undertaken. This is incentive compatible for the entrepreneur. The expected utility to the entrepreneur from holding a $\psi > 0$ when $VA \leq \bar{K}$ is less than or equal to the entrepreneur's expected utility from not undertaking the project. Consequently under the signaling equilibrium described in Proposition 4.2.1, all projects with $VA \leq \bar{K}$ are not undertaken.

4.2.3 WEAKLY INFORMATIONALLY CONSISTENT NASH EQUILIBRIA

So far in this section we have shown the existence and uniqueness of a Nash equilibrium that is informationally consistent. (Proposition 4.2.1). The signal provides accurate information as to the project's value (since $\mu[\psi(\mu)] = \mu$) and the price offers of investors equal this value. In this section we extend the notion of informationally consistent (or fully separating) equilibria.

DEFINITION: A set of price offers will be described as weakly informationally consistent (WINC) if the price paid to the entrepreneur given a signal, ψ, is equal to the average project value consistent with that signal.

For example, if the entrepreneur chooses the same level of the signal, $\bar{\psi}$, for all realizations of μ, and the single price offered by investors is

$$\bar{p} = \int_{\mu} VA(\mu;\psi)\, dF(\mu),$$

the average value of all available projects, $(\bar{\psi},\bar{p})$ is a weakly informationally consistent equilibrium. The example above is one of a pooling equilibrium. Alternatively, a WINC equilibrium may separate out some subsets of entrepreneurs, while the others are in one or more heterogeneous pools (such as in a partition equilibrium).

We next analyze the viability of weakly informationally consistent Nash equilibria. When no information is transferred (the case of pooling equilibrium) investors' valuation of the project will reflect the average value of all available projects. If the entrepreneur's reservation price is an increasing function of the project's valuation and the highest reservation price exceeds average value, the most valuable projects will not be traded. Because this lowers the average value of the remaining projects, more high quality projects will be withdrawn. Under our assumption that the number of projects with a value μ or more falls as μ increases, the 'adverse selection' process that we have noted may result in only the lowest quality projects being offered (Akerlof (1970)) and no weakly informationally consistent Nash equilibrium will exist.

We next consider a WINC equilibrium where the reservation prices of the highest quality projects exceed the average project value so that unravelling of the type described in the previous paragraph does not occur. We examine if such a WINC equilibrium is a Nash equilibrium. Riley (1979, p. 349) shows that under our assumption of a continuum of entrepreneur (or project) types μ, there is no weakly informationally consistent Nash eqilibrium price offer. He demonstrates that whenever heterogeneous projects (that is projects with different values of μ) are pooled, there exist alternative profitable offers by investors which draw away the higher type of entrepreneur from within the pool. As a result the average value of the projects left in the pool is lower than the initial price offer. Since the investors price offer $\bar{\underline{p}}$ is not equal to the average value of the projects in the pool that are signaled via $\bar{\Psi}$, $(\bar{\Psi},\bar{\underline{p}})$ is not a weakly informationally consistent Nash equilibrium.

4.2.4 SEQUENTIAL EQUILIBRIUM OF THE ENTREPRENEUR-INVESTORS GAME

We have demonstrated that the only Nash equilibrium of the game among the entrepreneur and investors is the informationally consistent (or fully separating) equilibrium identified in Proposition 4.2.1. We also showed that for any informationally consistent price function $\underline{p}(\psi)$, the level of signal ψ chosen by each entrepreneur type μ is unique. Our objective in this section is to examine if the informationally consistent Nash equilibrium of Proposition 4.2.1 is also a sequential equilibrium in the spirit of Kreps and Wilson (1982b).

As defined in Chapter 3, a sequential equilibrium is an assessment that is both consistent and sequentially rational. An assessment consists of a system of beliefs and the specification of strategies at each information set in the game. The substance of sequential rationality is that the strategy of each player starting from each information set must be optimal, starting from there according to some assessment over the nodes in the information set (beliefs) and the strategies of everyone else. An assessment will be sequentially rational if, taking the beliefs as fixed, no player prefers at any point to change his part of the strategy. We need to specify beliefs and strategies at all information sets both on and off the equilibrium path.

It is easy to see that the Nash equilibrium described in Proposition 4.2.1 is a sequential equilibrium. Before doing so, we note a technical point about applying the concept of sequential equilibrium to the game we have described. The definition of sequential equilibrium to the game we have described. The definition of sequential equilibrium in Kreps and Wilson (1982b) involved games with a finite number of moves in each information set which is not the case here. Our arguments, however, are based on the 'spirit' of the formal definitions.

Specification of beliefs is trivial in the game considered here. For the investor there is no unreached information set. Investor $j=1,2$ does not observe μ or ψ. His strategy is to choose a function $\underline{p}_j(\cdot)$ that maps all possible choices of ψ by the entrepreneur (which depend on μ) to $\underline{p}_j(\psi)$. Similarly, the entrepreneur's strategy is to choose ψ for all possible realizations of μ. The beliefs of each player are determined by the interaction of their strategies.

Formally, an assessment is sequentially rational if

(i) $$E[U(\tilde{W}_1;\mu,\sigma_E^*,\sigma_1^*,\sigma_2^*)] \geq E[U(\tilde{W}_1;\mu,\sigma_E,\sigma_1^*,\sigma_2^*)]$$
for all $\sigma_E \in \Sigma_E$ and for all $\mu \in M^A$

(ii) For all $i,j=1,2$, $i \neq j$

$$U_j(\sigma_j^*,\sigma_E^*,\sigma_i^*) \geq U_j(\sigma_j,\sigma_E^*,\sigma_i^*)$$
for all $\sigma_j \in \Sigma_j$

We now verify that the equilibrium described in Proposition 4.2.1 is a sequential equilibrium.

<u>PROPOSITION 4.2.2</u>: The strategies described in Proposition 4.2.1 and the beliefs given above constitute a sequential equilibrium that is

PROOF: Obvious. The requirements of sequential rationality are precisely the same as the conditions for a Nash equilibrium in Proposition 4.2.1. Q.E.D.

This completes our sytlized review of the Leland-Pyle (1977) model. In the next section, we introduce an auditor to produce information about the project's μ. The auditor's job is to verify information about μ contained in a prospectus issued by the entrepreneur. In Section 4.2.2, we argued that the 'signaling' activity that the entrepreneur must engage in imposes a loss in efficiency in an ex-ante classical sense. Our objective in the next section is to examine if employing an auditor improves efficiency in an ex-ante incentive sense relative to not employing an auditor and signaling via ψ alone. Obviously the efficiency of the equilibrium with auditing will depend on the audit fee that is paid to the auditor. The fee paid depends on the exogenously specified minimum utility level of the auditor. We assume that the audit fees are sufficiently small so that the potential for efficiency gains exist.

4.3 ENTREPRENEUR-INVESTORS-AUDITOR MODEL

To the structure of the entrepreneur-investors model of the previous section we add a set of possible actions for the auditor. The auditor takes actions under moral hazard. He can either choose to work, in which case we denote the auditor's choice of action as $\bar{c}$, or shirk, in which case we denote the auditor's act as c_s.[5] The auditor's function is to observe and verify the entrepreneur's 'type' μ. He then reports his assessment about μ to investors. The auditor's report $\hat{\mu} \in M^A$, where M^A is the set of all possible μ. The auditor's strategy is a measurable function from M^A to M^A. The audit report $\hat{\mu}$ is assumed to be observable by the entrepreneur, auditor and investors. The investors' decisions about the prices they should offer for shares in the project may now depend on ψ and $\hat{\mu}$.

The entrepreneur also chooses a scheme of transfers from himself to the auditor. The auditor's preferences are assumed representable by a von Neumann-Morgenstern utility function

$$H = Q(\cdot) - R(c)$$

$$Q'(\cdot) > 0, \; Q''(\cdot) < 0$$

$$R'(\cdot) > 0, \; R''(\cdot) > 0$$

where $Q(\cdot)$ denotes the auditor's utility for cash and $R(\cdot)$ denotes the non-pecuniary return. The auditor prefers more cash but dislikes effort. We continue to assume, as in Chapter 3, that payments to the auditor are unrelated to the audit report. The auditor is paid a fixed fee $g(c)$ for the level of audit c. The fee guarantees the auditor his minimum utility from alternative employment. We continue to assume that if the auditor shirks and is detected penalties are imposed on the auditor. Even so, with a fixed fee $g(c)$ the auditor may prefer to shirk rather than work. This is precisely the situation we model in

Chapter 3. There we demonstrate that in a multi-period game with incomplete information, reputation and penalty considerations may cause the auditor to work even though shirking is optimal in a single period game. In this chapter, for the sake of expositional clarity and in order to focus on the adverse selection issues, we consider only a single period game. In a multi-period game we could have shown as in Chapter 3 that reputation and penalty effects provide the auditor with the necessary incentives to work hard and report truthfully. For the rest of this chapter, however, we assume that sufficient incentives exist for the auditor in the single period (of the multi-period game in which he is engaged) to take the action choice c. That is, we do not model moral hazard on the auditor's part. In a larger, more complete model of the type in Chapter 3 this assumption is not important.

The objective of this section is to examine if employing the auditor can improve the ex-ante incentive efficiency in the game between the entrepreneur and investors. Since the only sequential equilibrium in the game among the entrepreneur and investors is an informationally consistent sequential equilibrium we shall only consider an informationally consistent equilibrium. In the next subsection we describe the structure of the game among the entrepreneur, auditor and investors.

4.3.1 STRUCTURE OF THE ENTREPRENEUR-AUDITOR-INVESTORS GAME

At the start of the game, the entrepreneur obtains information about $\mu \in M^A \subseteq R$. The investors' information about μ (as encoded in $F(\mu)$), and $\tilde{v}$, the randomness in the project's return, is exactly as before. Each investor $j=1,2$ offers a price schedule $\underline{p}_j(\cdot)$ as a measurable function of ψ and $\hat{\mu}$ (the latter if the auditor is employed to issue a report). After observing μ and the investors' pricing schedule, the entrepreneur <u>may</u> choose to employ the auditor to take action c and issue an audit report $\hat{\mu}$. We may sometimes refer to c as the level of audit provided by the auditor. It is common knowledge that the audit report is subject to random audit errors so the audit does not reveal μ perfectly. In particular, the audit report $\tilde{\hat{\mu}}$ is a random variable distributed on $[\mu, \mu + u(c)]$, where $u(c)$ denotes the maximum amount by which the audit report can overstate μ as a result of audit errors when the auditor works hard and takes the action choice c. The probability density function of $\tilde{\hat{\mu}}|\mu$ is denoted by $\bar{h}(\tilde{\hat{\mu}}|\mu)$.[6] If the entrepreneur decides to employ the auditor, the entreprenuer chooses ψ after $\hat{\mu}$ is announced. ψ and $\hat{\mu}$ are common knowledge. Furthermore, since sufficient incentives exist for the auditor to provide a level of audit c, the maximum extent of potential audit error $u(c)$ is also common knowledge. If the entrepreneur prefers not to employ the auditor he simply chooses ψ. Assuming competitive capital markets it can be shown as before that each investor's valuation of the project VA which in general is a function of ψ and $\hat{\mu}$ is given by

$$VA(\mu;\hat{\mu},\psi) = \frac{1}{(1+r)} [\mu(\hat{\mu},\psi) - \lambda] \tag{4.11}$$

where λ is, as before, the market's adjustment for the risk of the project.

4.3.2 STRATEGIES AND PAYOFFS OF THE ENTREPRENEUR-AUDITOR-INVESTORS GAME

Investor j chooses the price $\underline{p}_j$ to offer for the shares as a function of the audit report $\hat{\mu}$ (if the auditor is employed) and ψ.

DEFINITION: A pure strategy for investor j=1,2 is the choice of a measurable function $\underline{p}_j(\cdot)$ mapping each $\hat{\mu}$ and each ψ that the investor may observe into a price offer $\underline{p}_j$, that is

$\sigma_j : C^A \to \underline{p}_j(\cdot)$ where

$\underline{p}_j(\cdot) : [0,1] \to P_j^A$ if the auditor is not employed

$\underline{p}_j(\cdot) : M^A x[0,1] \to P_j^A$ if the auditor is hired and issues a report.

C^A is the common knowledge information set.

The set of pure strategies for investor j, j=1,2 is denoted by Σ_j.

After observing μ, $\underline{p}_1(\cdot)$ and $\underline{p}_2(\cdot)$ the entrepreneur decides whether to employ the auditor (that is, obtain $\hat{\mu}$). The entrepreneur's decision can be represented by a measurable function from $M^A \times P_1^A \times P_2^A \to \{0,1\}$ where 0 denotes the entrepreneur does not employ the auditor and 1 denotes he does. The entrepreneur then chooses ψ and ν.

DEFINITION: A pure strategy σ_E for the entrepreneur is a measurable function mapping each μ and $\hat{\mu}$ given $\underline{p}_1(\psi)$ and $\underline{p}_2(\psi)$ into a choice of auditor, ψ and ν, that is

$$\sigma_E : M^A \times M^A \times P_1^A \times P_2^A \to \{0,1\} \times [0,1] \times [0,1].$$

The set of pure strategies for the entrepreneur is denoted Σ_E.

We do not emplicitly write down the strategic choices for the auditor since, as noted, we assume sufficient incentives exist for the auditor to work rather than shirk.

The entrepreneur sells his shares to whichever investor offers the highest price $\underline{p}$. The entrepreneur's payoffs are slightly modified relative to the previous section if the auditor is employed. The budget constraint must now include the cost of the audit g(c), and we have

$$\bar{W}_0 + (1-\psi)\,\underline{p} - \bar{R} - g(c) - \nu V_M - Y = 0 \qquad (4.12)$$

The outcome $\tilde{W}_1$ is given as before by

$$\tilde{W}_1 = \psi(\mu+\tilde{v}) + \nu\tilde{M} + (1+r)Y$$

Substituting for Y from (4.12) yields

$$\tilde{W}_1 = \psi[\mu + \tilde{v} - (1+r)\underline{p}] + \nu[\tilde{M} - (1+r)V_M] + [W_0 - g(c)](1+r) + (1+r)\underline{p} \quad (4.13)$$

where $W_0 = \bar{W}_0 - \bar{R}$.

If the entrepreneur decides not to employ the auditor, the entrepreneur's payoff is precisely as defined in the previous section and can be derived by setting g(c) equal to zero in equation (4.13).

If the auditor is employed by the entrepreneur, the entrepreneur's expected utility is given by

$$E[U(\tilde{W}_1;\mu,\sigma_E,\sigma_1,\sigma_2)] = G\{\psi[\mu - (1+r)\underline{p}] + \nu[\bar{M} - (1+r)V_M] + (1+r)(W_0-g(c))$$

$$+ (1+r)\underline{p} - \frac{a}{2}[\psi^2\sigma_V^2+\nu^2\sigma_M^2+2\psi\nu \operatorname{cov}(\tilde{V},\tilde{M})]\} \quad (4.14)$$

Investor j's payoff $j=1,2 \quad i\neq j$ is given by

$U_j(\sigma_j,\sigma_E,\sigma_i) =$

$(1-\psi)[VA(\hat{\mu};\mu,\psi) - \underline{p}_j(\hat{\mu},\psi)]$,	if the auditor is hired and the investor acquires shares in the project,
$(1-\psi)[VA(\mu;\psi) - \underline{p}_j(\psi)]$	if the auditor is not employed and the investor gets shares in the project
0	Otherwise

4.3.3 NASH EQUILIBRIUM OF ENTREPRENEUR-AUDITOR-INVESTORS GAME

DEFINITION: $(\sigma_E^*,\sigma_1^*,\sigma_2^*)$ is a pure strategy Nash equilibrium if

(i) $E[U(\tilde{W}_1;\mu,\sigma_E^*,\sigma_1^*,\sigma_2^*)] \geq E[U(W_1;\mu,\sigma_E,\sigma_1^*,\sigma_2^*)]$

for all $\sigma_E \in \Sigma_E$, for all μ, $\hat{\mu} \in M^A$.

(ii) for $i,j=1,2$, $i \neq j$

$$U_j(\sigma_j^*,\sigma_E^*,\sigma_i^*) \geq U_j(\sigma_j,\sigma_E^*,\sigma_i^*)$$

for all $\sigma_j \in \Sigma_j$

This is just an adaptation of the earlier definition of Nash equilibrium.

A price function $\underline{p}(\psi)$ is defined to be informationally consistent exactly as in the previous section, if the auditor is not employed. The definition of an informationally consistent price function is modified slightly in the event the auditor is employed and issues an audit report $\hat{\mu}$. Basically we need to condition on the audit report as well.

A price function $\underline{p}(\hat{\mu},\psi)$ is informationally consistent if

(i) Entrepreneurs choose ψ to maximize $E[U(\tilde{W}_1|\psi,\underline{p}(\hat{\mu},\psi)]$

and (ii) $\underline{p}(\hat{\mu},\psi) = VA(\mu;\hat{\mu},\psi)$.

Before we present a Nash equilibrium in the game, we briefly discuss the entrepreneur's strategic choice regarding whether to employ the auditor. If $\mu > (1+r)\bar{K} + \lambda$, the entrepreneur can signal μ via ψ as in the previous section. In such a case, we denote the entrepreneur's expected utility by $E_{\tilde{V},\tilde{M}}[U(\tilde{W}_1)|\psi]$.

If instead the entrepreneur chooses to employ the auditor he incurs an audit fee g(c). The benefit of doing so is that the entrepreneur can now condition his choice of ψ on the audit report $\hat{\mu}$. Since $\hat{\mu}$ is a random variable (due to random audit errors) the entrepreneur computes his expected utility conditional on all possible realizations of $\hat{\mu}$ and the corresponding choice of ψ. We denote the entrepreneur's expected utility by $E_{\tilde{\hat{\mu}}|c}[E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\hat{\mu},\psi]]$. It will be optimal for the entrepreneur to hire the auditor for values of μ that satisfy.

$$E_{\tilde{\hat{\mu}}|c}\left[E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\hat{\mu},\psi]\right] > E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\psi] \qquad (4.15)$$

We assume (4.15) is satisfied for some μ. (4.15) implicitly presumes that audit costs are sufficiently small.

In Proposition 4.3.1 we describe an informationally consistent Nash equilibrium for the game we have described.

PROPOSITION 4.3.1

The following strategies of the entrepreneur and each investor j constitute an informationally consistent Nash equilibrium in the game.

Investor j's Strategy

(i) If the auditor is not employed and $\psi = 0$, $\underline{p}_j(\psi) = 0$.

(ii) If the auditor is not employed and $\psi > 0$, each investor j offers a price function

$$\underline{p}_j(\psi) = VA(\mu;\psi) = \frac{1}{(1+r)}[\mu(\psi) - \lambda] \qquad (4.17a)$$

where $\mu(\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + (1+r)\bar{K} + \lambda \qquad (4.5)$

(iii) If the auditor is employed, each investor j offers a price schedule

$$\underline{p}_j(\hat{\mu},\psi) = VA(\mu;\hat{\mu},\psi) = \frac{1}{(1+r)}[\mu(\hat{\mu},\psi) - \lambda] \qquad (4.17b)$$

where $\mu = \mu(\hat{u},\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + \max(\hat{u}-u(c),\ (1+r)\bar{K} + \lambda)$ (4.16)

Entrepreneur's Strategy

(i) After observing μ, $\underline{p}_1(\cdot)$, $\underline{p}_2(\cdot)$

if $\mu \leq (1+r)\bar{K} + \lambda$, the entrepreneur does not undertake the project.

if $(1+r)\bar{K} + \lambda < \mu < \mu_0$, the entrepreneur does not hire the auditor but chooses ψ using $\mu = \mu(\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + (1+r)\bar{K} + \lambda$

if $\mu \geq \mu_0$, the entrepreneur employs the auditor and after observing the audit report $\hat{u}$, chooses ψ using

$$\mu = \mu(\hat{u},\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + \max(\hat{u}-u(c),(1+r)\bar{K} + \lambda)$$

where μ_0 is the value of μ at which

$$E_{\tilde{u}|c}\left[E_{\tilde{v},\tilde{M}}[U(W_1)|\hat{u},\psi]\right] = E_{\tilde{v},\tilde{M}}(U(\hat{W}_1)|\psi]$$

PROOF: See Appendix 4.

Proposition 4.3.1 demonstrates the existence of a Nash equilibrium that is informationally consistent. In other words the equilibrium is a fully separating equilibrium in which $\hat{u}$ and ψ signal μ. Moreover, we have demonstrated that employing the auditor improves ex-ante incentive efficiency for all values of $\mu \geq \mu_0$. To see this note that the entrepreneur's expected utility from employing the auditor is greater than his expected utility from not employing the auditor for all values of $\mu \geq \mu_0$. On the other hand, investors are as well off as before. In any informationally consistent equilibrium, the price investors pay for the shares equals the monetary value of the shares.

As in the previous section, the equilibrium we have described is unique. Any other price schedule offered by investor j, j=1,2 is not an equilibrium in the sense that there exists an alternative price schedule that will make the investor better off (see also Riley 1979, p. 348 and Riley, 1975).

4.3.4 WEAKLY INFORMATIONALLY CONSISTENT NASH EQUILIBRIA

So far in this section we have shown the existence and uniqueness of a Nash equilibrium that is informationally consistent. The objective of this section is to examine if there exists a weakly informationally consistent Nash equilibrium. We continue to assume that the cummulative probability distribution of μ and $\hat{\mu}$ is concave. The arguments of Section 4.2.3 can then easily be applied to the setting we have described in this section. It follows that there is no weakly informationally consistent Nash equilibrium price schedule.

4.3.5 SEQUENTIAL EQUILIBRIUM OF THE ENTREPRENEUR-AUDITOR-INVESTORS GAME

In this section we shall examine if the equilibrium of Proposition 4.3.1 is also a sequential equilibrium in the spirit of Kreps and Wilson (1982b). The crux of the requirement of a sequential equilibrium is to specify each investor's beliefs both on and off the equilibrium path. If the auditor is not employed the beliefs and strategies that constitute a sequential equilibrium are exactly as specified in Section 4.2.4. Even when the auditor is hired the belief specification is trivial and derived from the strategies of the players. Investors do not observe μ, $\hat{\mu}$ or ψ. The investors' price function $\underline{p}_j(\hat{\mu},\psi)$ specifies the price to be paid for all possible observations of $\hat{\mu}$ and ψ. Therefore, there are no unreached information sets for the investor. After observing μ and the price schedules, the entrepreneur obtains $\hat{\mu}$ and chooses ψ to maximize his expected utility. The beliefs of each player are determined by the interaction of their strategies.[7]

Formally, an assessment is sequentially rational in the game if

(i) $$E[U(\tilde{W}_1;\mu,\sigma_E^*,\sigma_1^*,\sigma_2^*)] \geq E[U(\tilde{W}_1;\mu,\sigma_E,\sigma_1^*,\sigma_2^*)]$$

$$\text{for all } \sigma_E \in \Sigma_E \text{ and for all } \mu,\ \hat{\mu} \in M^A$$

(ii) (A) if the auditor is employed,

for $j=1,2$ $i \neq j$

$$U_j(\sigma_j^*,\sigma_E^*,\sigma_i^*) \geq U_j(\sigma_j,\sigma_E^*,\sigma_i^*)$$

for all $\sigma_j \in \Sigma_j$

(iii) (B) if the auditor is not employed,

for $j=1,2,\ i \neq j$

$$U_j(\sigma_j^*, \sigma_E^*, \sigma_i^*) \geq U_j(\sigma_j, \sigma_E^*, \sigma_i^*)$$

for all $\sigma_j \in \Sigma_j$

Subject to the technical point mentioned earlier in Section 4.2.4, we can easily verify that the equilibrium described in Proposition 4.3.1 is a sequential equilibrium.

<u>PROPOSITION 4.3.4</u> The strategies described in Proposition 4.3.1 and the beliefs given above constitute a sequential equilibrium that is informationally consistent.

<u>PROOF</u>: Obvious. The requirements of sequential rationality are precisely the same as the requirements in Proposition 4.3.1. Q.E.D.

We have therefore verified an informationally consistent sequential equilibrium in the entrepreneur-auditor-investors game. Also note that since there is no weakly informationally consistent Nash equilibrium, it follows that there is no sequential equilibrium which is weakly informationally consistent.

We graphically compare the equilibrium with auditing with the signaling equilibrium without auditing. We assume that $u(c) < (1+r)\, g(c)$. This assumption simplifies our description, in that, whenever the auditor is chosen, $\hat{\mu} - u(c)$ is always greater than $R(1+r) + \lambda$. Figure 4.1 compares the functions relating ψ to μ in the equilibria with and without auditing for particular realizations of $\hat{\mu}_1$ and $\hat{\mu}_2$.

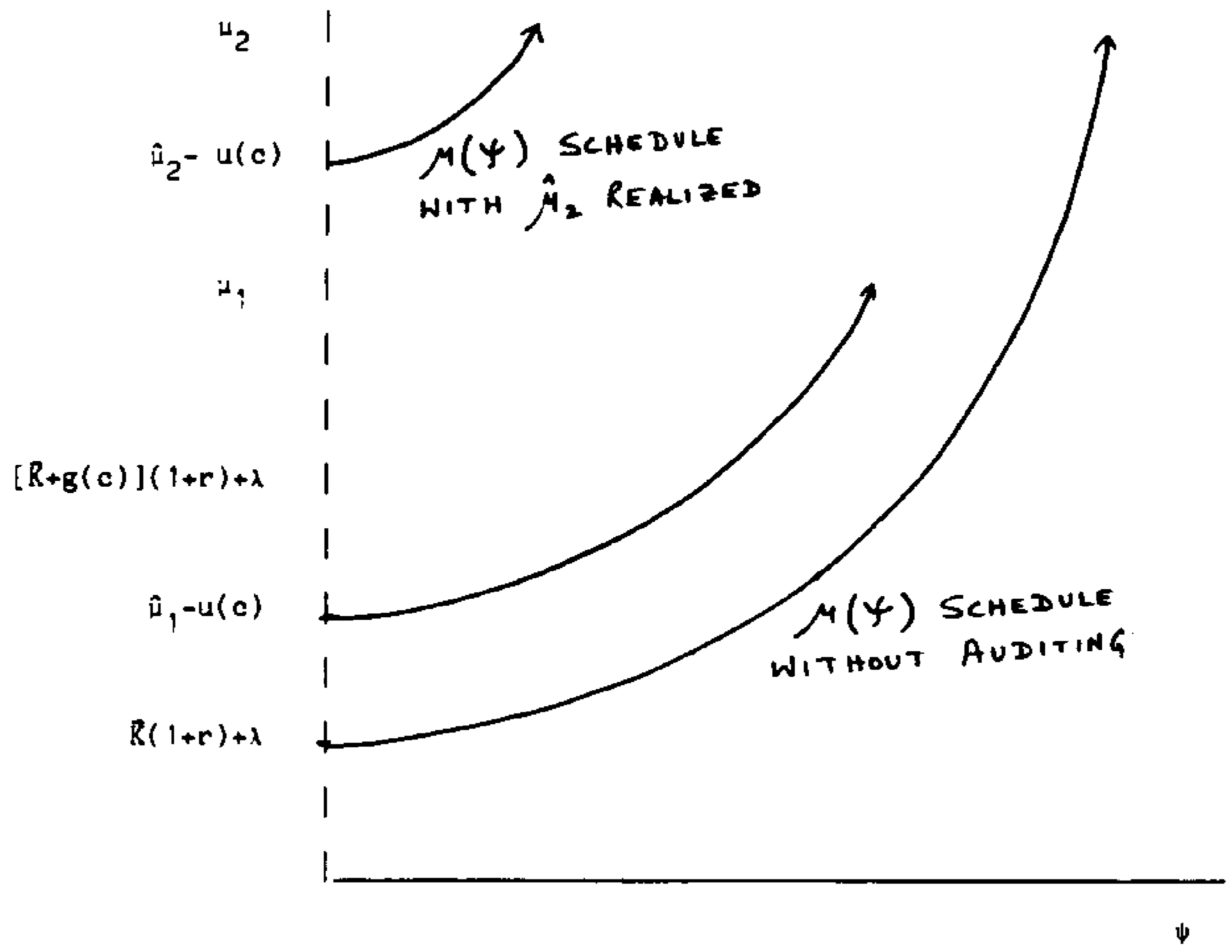


FIGURE 4.1

Figure 4.2 compares the entrepreneur's expected utility as a function of μ in the signaling equilibrium with auditing and the signaling equilibrium without auditing.

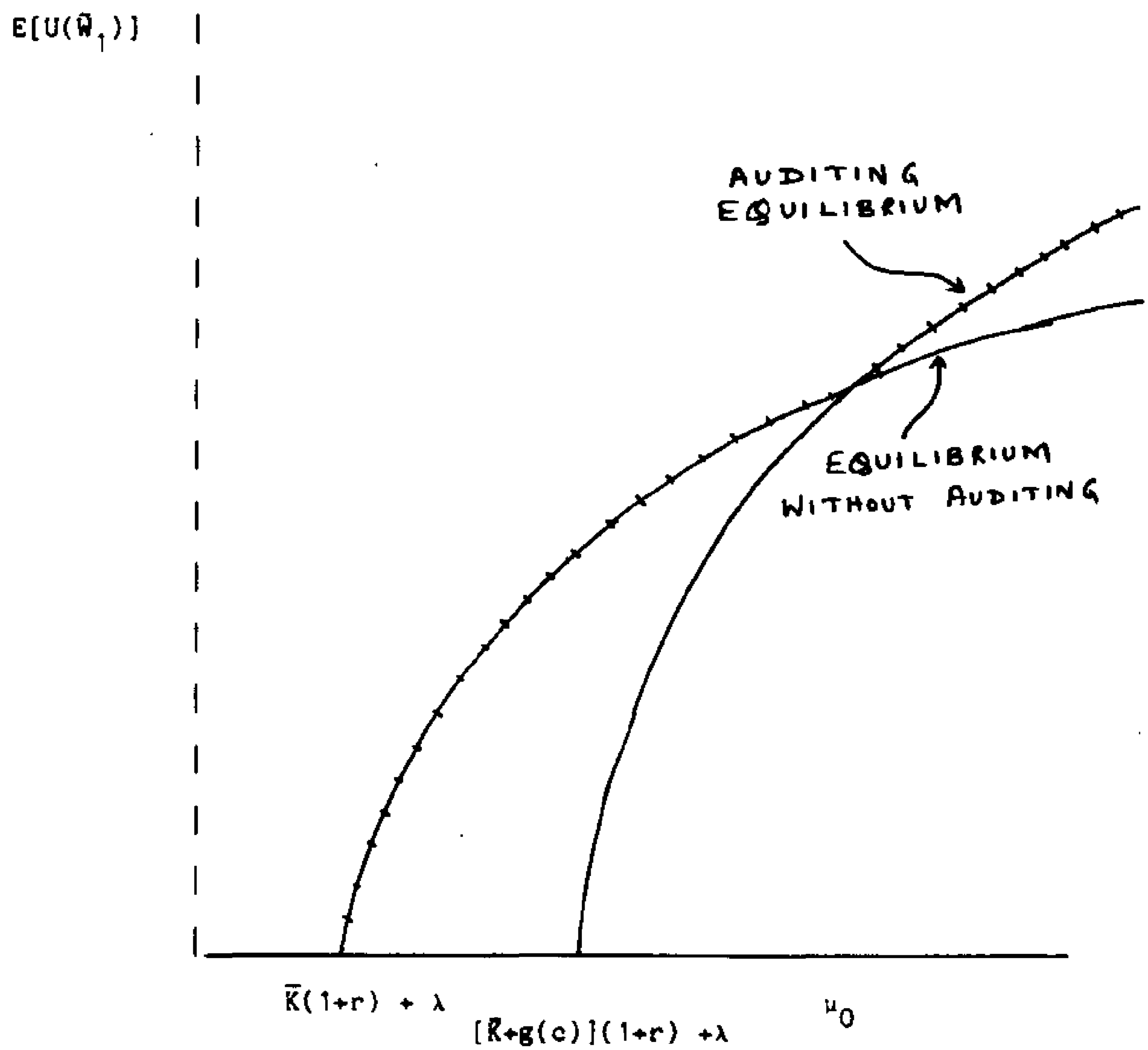


FIGURE 4.2

The crossed lines indicate that up to μ_0 the entrepreneur signals μ via ψ alone whereas for $\mu \geq \mu_0$ the auditing equilibrium dominates and is ex-ante incentive efficient.

4.3.6 NECESSARY CONDITION FOR AUDITING TO BE EFFICIENT

In Proposition 4.3.4 we demonstrated that if $\hat{u}$ is distributed over $[\mu, \mu + u(c)]$ there exists an informationally consistent sequential (Nash) equilibrium in which the equilibrium with auditing is more efficient than the equilibrium without auditing for values of μ greater than μ_0. This follows since $E_{\tilde{\hat{u}}|c}[E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1|\hat{u},\psi)]] > E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\,\psi]$ for $\mu > \mu_0$ and investors are no worse off (the price they pay equals the value of their shares). In other words $\hat{u}$ distributed on $[\mu, \mu+u(c)]$ is sufficient for the auditing equilibrium to be efficient for $\mu > \mu_0$. In this section we show that a necessary condition for an informationally consistent equilibrium with auditing to be more efficient than the equilibrium without auditing is that $\hat{u}$ has moving support.

To see this first observe that in the equilibrium of Proposition 4.2.1 (the signaling equilibrium without auditing), the fraction of equity in the project held by the entrepreneur is a function of μ but is independent of the investors' prior distribution of μ over M^A. Informational consistency demands that investors correctly identify μ for all $\mu \in M^A$, whatever be the investors' prior distribution of μ. We require $\underline{p}(\psi) = VA(\mu;\psi)$ and $\mu(\psi^*(\mu)) = \mu$ for all $\mu \in M^A$ irrespective of the probability distribution of μ over M^A.[8]

Consider the case where the audit report $\hat{\mu}$ affects the investors' prior distribution of μ but does not alter the support of the distribution. It follows from our earlier comment that the fraction of equity in the project held by the entrepreneur in the equilibrium with auditing is the same as that held by the entrepreneur in the equilibrium without auditing. It is then clearly inefficient to employ the auditor and incur the audit cost g(c). We therefore conclude that for the auditing equilibrium to be efficient, the audit report must alter the support of the investors' probability distribution about μ. Indeed, we demonstrate in Proposition 4.3.1 that the optimal $\psi^*(\mu)$ does depend on the support of the distribution of μ. The fraction of equity, ψ_A, that the entrepreneur retains in the auditing equilibrium (to communicate the actual μ to investors) is strictly less than the fraction of equity ψ_S that the entrepreneur retains in the absence of auditing. The smaller holding of equity in the project results in an increase in the entrepreneur's expected utility. This benefit must be balanced against the disutility of incurring higher audit costs.[9]

Thus far we have focused on necessary and sufficient conditions for employing an auditor. In the next section we introduce a second auditor who provides a level of effort, or audit quality, $\bar{c}$. Our objective is to examine the entrepreneur's strategic decision regarding the choice of auditor.

4.4 AUDITOR CHOICE IN THE ENTREPRENEUR-AUDITOR-INVESTORS GAME

In this section we consider a simple extension of the above game by introducing a second auditor. The second auditor provides a level of effort or audit quality, $\bar{c}$. Given μ and $\bar{c}$, $\tilde{\hat{\mu}}$ is a random variable distributed on $[\mu, \mu + u(\bar{c})]$. We assume that the maximum audit error $u(\bar{c})$ when audit quality $\bar{c}$ is chosen is less than $u(c)$ the maximum audit error when c is chosen, that is, we assume $\bar{c}$ to be a more precise audit. If $\bar{c}$ is chosen, $\tilde{\hat{\mu}} - u(\bar{c})$ is a random variable distributed on $[\mu - u(\bar{c}), \mu]$ with a probability density function $\bar{f}$ and a probability distribution function $\bar{F}$. The density and distribution function of $\tilde{\hat{\mu}} - u(c)$ are denoted by f_1 and F_1 respectively. We also assume that

$$\bar{F}(\hat{\mu} - u(c)) \le F_1(\hat{\mu} - u(c)) \text{ for all } \hat{\mu} - u(c) \in [\mu - u(c), \mu].$$ [10]

That is, the probability density $\bar{f}$ is at least as large as f_1 in the sense of first-order stochastic dominance (FSD).[11] This assumption implies the probability of $\hat{\mu} - u(c)$ being 'closer' to μ is larger for auditors of quality $\bar{c}$ than for auditors of quality c.

We are now in a position to state and prove Proposition 4.4.1.

PROPOSITION 4.4.1 If the audit cost $g(\cdot)$ is the same for audit qualities c and $\bar{c}$,[12] then under our assumptions about the support and distribution of $\bar{c}$ relative to c, the entrepreneur prefers to employ an auditor of quality $\bar{c}$ rather than c in equilibrium.

PROOF: See Appendix 4.

The intuition is that rational investors will discount more heavily, the values of $\hat{\mu}$ reported by auditors of lower audit quality. As indicated earlier, we can weaken the first-order stochastic dominance condition to require instead second-order stochastic dominance, that is,

$$\int_{\mu - u(c)}^{\hat{\mu} - u(c)} \bar{F}(y)dy \le \int_{\mu - u(c)}^{\hat{\mu} - u(c)} F_1(y)dy$$

for all $\hat{\mu} - u(c) \in [\mu - u(c), \mu]$.

We can also relax the condition that the audit cost $g(\cdot)$ is the same for audit qualities c and $\bar{c}$. Proposition 4.4.1 holds even if audit costs are assumed to increase with audit quality provided the benefit from shifting the cumulative distribution of $\hat{\mu} - u(c)$ exceeds the increase in audit costs.

In general, the choice of auditor will depend on the audit cost g(c) and the "productive" effect of audit quality as captured by the cumulative distribution of $\hat{\mu} - u(c)$. Entrepreneurs will choose auditors of higher quality until the marginal benefit from shifts in $\hat{\mu} - u(c)$ equals the marginal audit cost. Therefore, if the audit cost increases sharply with increases in c relative to the benefit from shifts in $\hat{\mu} - u(c)$, entrepreneurs will choose low quality auditors. If, on the other hand, the differential audit cost of choosing higher quality auditors is small, entrepreneurs will choose auditors of high quality.

Introducing a second auditor raises the question of using the choice of auditor c as a signal of the entrepreneur's private information μ. The necessary Spencian condition is that audit costs at the margin be negatively correlated with μ, that is, $\frac{d^2g}{d\mu dc} < 0$. The condition implies that marginal audit cost decreases with increases in μ. We doubt whether this condition will be satisfied in the auditing context. Instead, we expect marginal audit cost to be uncorrelated with μ and signaling via c will not be feasible.

4.5 COMPARATIVE STATICS

We now examine some comparative statics properties of the equilibrium.

PROPOSITION 4.5.1 An increase in the specific risk $\bar{Z}$ of the project or risk aversion a of the entrepreneur will reduce the entrepreneur's equilibrium equity position $\psi^*(\hat{u},\mu)$ for any value of μ at which the project is undertaken.

PROOF: For any fixed μ, we have $\mu(\hat{u},\psi) = \mu$ that is

$$-a\bar{Z}[\log(1-\psi)+\psi] + \max\ (\hat{u}-u(c),\ \bar{R}(1+r)+\lambda) = \mu$$

Differentiating with respect to $a\bar{Z}$, we have

$$-[\log(1-\psi)+\psi] + \frac{a\bar{Z}\psi}{1-\psi}\frac{d\psi}{da\bar{Z}} = 0$$

that is

$$\frac{d\psi}{da\bar{Z}} = \frac{(1-\psi)[\log(1-\psi)+\psi]}{a\bar{Z}\psi} < 0$$

since $[\log(1-\psi)+\psi] < 0$ for $\psi > 0$.

PROPOSITION 4.5.2 As the audit fee g(c) increases, the entrepreneur is strictly worse off for all values of $\mu > \mu_0$. This means that increases in g(c) decreases the ex-ante incentive efficiency of employing the auditor.

PROOF $$\frac{\partial E_{\tilde{\hat{\mu}}|c}[E[U(\tilde{W}_1|\hat{\mu},\psi)]]}{dg(c)} = \begin{cases} -(1+r)\ G'(\cdot)\cdot g'(c_1) < 0 & \text{for } \mu > \mu_0 \\ 0 & \text{for } \mu \leq \mu_0 \end{cases}$$

As is evident from figure 4.3, the entrepreneur's expected utility is not affected for values of $\mu \leq \mu_0$. The entrepreneur's expected utility is strictly lower for values of $\mu > \mu_0$, since the benefit from auditing is unchanged but the cost of auditing has increased. The investors' welfare is unaffected by changes in the audit fee, g(c).

PROPOSITION 4.5.3 An increase in audit error u(c) decreases the entrepreneur's expected utility for all $\mu > \mu_0$.

PROOF $$\frac{\partial E_{\tilde{\hat{\mu}}|c}[E[U(\tilde{W}_1|\hat{\mu},\psi)]]}{du(c_1)} = \begin{cases} -G'(\cdot).\ (1-\psi) < 0 & \text{for } \mu > \mu_0 \\ 0 & \text{for } \mu \leq \mu_0 \end{cases}$$

As in Proposition 4.5.2, the entrepreneur's expected utility is unaffected for values of $\mu \leq \mu_0$. For $\mu > \mu_0$ (the region where the auditing equilibrium dominates) the entrepreneur is strictly worse off because an increase in the audit error increases the expected ψ that the entrepreneur must hold to signal μ. This reduces the ex-ante incentive efficiency of employing the auditor. This follows since the investors welfare is unaffected by changes in u(c).

4.6 CONCLUSION

In this chapter we considered a situation in which an entrepreneur with superior information about a project's quality wishes to issue shares to outside investors. We demonstrate existence of a sequential equilibrium of the entrepreneur-auditor-investors game in which employing the auditor is ex-ante incentive efficient. In particular, it is efficient to employ the auditor when the market value of the project is substantially greater than project cost. Employing the auditor results in the entrepreneur not having to hold a disproportionately large amount of shares in his own project in order to convince the market about the project's value.

The sequential equilibrium is informationally consistent. In an informationally consistent equilibrium, the signal perfectly reveals the project's value and each investor offers a price equal to the project's value. We argue that there exists no (weakly) informationally consistent (pooled or partially pooled) equilibrium.

In Section 4.4 we introduced multiple auditors and examined the entrepreneur's strategic audit choice decision. We show that if variation in audit fees across auditors of varying quality (defined in terms of audit procedures, sampling techniques, audit judgement, etc.) is small, all entrepreneurs will choose the highest level of audit quality. In other words, heterogeneous entrepreneurs choose the same (type of) auditor. This provides a rationale for the dominance of high quality premium audit firms. It is also consistent with the observation that firms tend to switch to a particular 'type' of auditor (Big Eight accounting firms) when they go public.

Throughout our analysis, we had assumed that the distribution of $\hat{\mu} - u(\bar{c})$ stochasticaly dominates $\hat{\mu} - u(c)$ for all $\bar{c} > c$ and for all μ. An alternative assumption is that $\hat{\mu} - u(\bar{c})$ stochastically dominates $\hat{\mu} - u(c)$ for larger values of μ but that $\hat{\mu} - u(c)$ stochastically dominates $\hat{\mu} - u(\bar{c})$ for smaller values of μ. The intuition is that for small values of μ, the audit report $\hat{\mu}$ is more likely to report a larger value when c is chosen than when $\bar{c}$ is chosen. The analysis in such a case is a little more complicated. If entrepreneurs with low realizations of μ were to choose c, it would serve as a signal to the market that a low μ has in fact been observed. (Recall that for high realizations of μ, the entrepreneur will choose $\bar{c}$). Consequently, even under the assumptions described above, entrepreneurs with low realizations of μ may choose $\bar{c}$ and 'pool' with entrepreneurs with high realizations of μ so as not to reveal a low μ to the market. This will once again result in all entrepreneurs choosing the higher level of audit quality $\bar{c}$.

APPENDIX 4

Proof of Proposition 4.2.1

The steps in the proof of Proposition 4.2.1 are as follows.

(1) We first verify that given informational consistency, that is, $\underline{p}_j(\psi) = VA(\mu;\psi)$ and $\mu(\psi) = \mu$, the entrepreneur maximizes expected utility by choosing ψ (and ν) according to equation (4.5).

(2) We then argue that since investors are unaffected by the amount of ψ retained by the entrepreneur, each investor's equilibrium price offer is based on the $\mu(\psi)$ schedule which maximizes the entrepreneur's utility.

(3) It also follows that the constant of integration C_0 must equal $(1+r)\bar{R} + \lambda$. If one investor offers a C_0 less than $(1+r)\bar{R} + \lambda$, the second investor can make positive profits by choosing a slightly higher C_0 and offering a higher price $\underline{p}(\psi)$.

(4) We finally verify that after inferring μ, the price offer $\underline{p}(\psi) = VA(\mu;\psi)$ for all μ.

We start by showing that for $\mu > (1+r)\bar{R} + \lambda$ equation (4.5) maximizes the entrepreneur's expected utility given the investor's strategy $\underline{p}(\psi) = VA(\mu;\psi)$. Notice that in equilibrium, investors use ψ to infer μ correctly and offer a price $\underline{p}(\psi)$ equal to the project's value. If, for all ψ, $\mu(\psi)$ were greater than the actual μ of an entrepreneur retaining ψ, investors would on average pay a price $\underline{p}(\psi)$ greater than the value of the project $VA(\mu)$ so that $\underline{p}(\psi) \neq VA(\mu;\psi)$. If, on the other hand, $\mu(\psi)$ consistently underestimated the entrepreneur's actual μ, $\underline{p}(\psi)$ will be less than the value of the project $VA(\mu;\psi)$.

VERIFICATION THAT THE ENTREPRENEUR'S STRATEGY IS A BEST RESPONSE

To see that the entrepreneur is playing optimally we show that choosing ψ according to (4.5) is necessary and sufficient to $\underset{\psi,\nu}{\text{Maximize}}\ E[U(\tilde{W}_1);\ \mu,\ \sigma_E^*,\ \sigma_1^*,\ \sigma_2^*]$ given the investors' strategies. The entrepreneur's problem is as follows:

$$\text{Maximize } \psi[\mu - (1+r)\underset{\sim}{p}(\psi)] + \nu[\bar{M} - (1+r)V_M] + (1+r)\ W_0 + (1+r)\ \underset{\sim}{p}(\psi)$$

$$- \frac{a}{2}\ [\psi^2\sigma_v^2 + \nu^2\sigma_M^2 + 2\ \psi\nu\ \text{cov}(\tilde{v},\tilde{M})]$$

where $\underset{\sim}{p}(\psi) = VA(\mu;\psi) = \frac{1}{(1+r)}\ [\mu(\psi) - \lambda]$

First-order necessary conditions yield

$$[\mu - \mu(\psi) + \lambda] + (1-\psi)\mu_\psi - a\psi\sigma_v^2 - a\nu\text{cov}(\tilde{v},\tilde{M}) = 0 \qquad (4.6)$$

$$[\bar{M} - (1+r)V_M] - a\psi\text{cov}(\tilde{v},\tilde{M}) - a\nu\sigma_M^2 = 0 \qquad (4.7)$$

Substituting for $a\nu$ from (4.7) and using the condition $\mu(\psi^*(\mu)) = \mu$ enables us to derive the following differential equation for the valuation schedule $\mu(\psi)$.

$$(1-\psi)\mu_\psi = a\psi\bar{Z} \qquad (4.8)$$

where $\bar{Z} = \dfrac{\sigma_v^2\sigma_M^2 - [\text{cov}(\tilde{v},\tilde{M})]^2}{\sigma_M^2}$

The solution to the differential equation (4.8) is a family of functions

$$\mu(\psi) = -a\bar{Z}[\log(1-\psi) + \psi)] + C_0$$

It can easily be verfied that schedules in the family satisfy the second-order conditions for expected utility maximization. Leland and Pyle (1977) use further equilibrium arguments to specify the appropriate boundary condition $C_0 = \bar{K}(1+r) + \lambda$.

Therefore, choosing ψ according to

$$\mu(\psi) = -a\bar{Z}[\log(1-\psi)+\psi] + (1+r)\bar{K} + \lambda;\ \psi > 0, \qquad (4.5)$$

is optimal.

It is easy to verify that given the investors strategies σ_1^*, σ_2^*,

$$E[U(\tilde{W}_1);\mu,\psi > 0,\ \sigma_1^*,\ \sigma_2^*] \le E[U(\tilde{W}_1);\mu,\psi = 0,\ \sigma_1^*,\ \sigma_2^*]$$

for all $\mu \le (1+r)\ \bar{K} + \lambda$. Therefore, entrepreneurs with $\mu \le (1+r)\ \bar{K} + \lambda$ prefer not to undertake the project. Equation (4.5) suggests that for $\mu > (1+r)\bar{K} + \lambda$, $\mu(\psi)$ is a strictly increasing function of ψ. Consequently, investors can invert it to precisely determine the entrepreneur's μ and the project's value. Investors read higher entrepreneurial ownership as a signal of a more favorable project. In turn, entrepreneurs choose a higher fraction of ownership in projects with a higher μ.

VERIFICATION THAT EACH INVESTOR j'S STRATEGY IS A BEST RESPONSE

We next verify that each investor j will offer a price schedule $\underline{p}(\psi) = VA(\mu;\psi)$ with $\mu(\psi)$ given by equation (4.5). Given the entrepreneur's strategy investors can infer the entrepreneur's actual value of μ and the project value $VA(\mu;\psi)$ for all $\psi > 0$.

If the price $\underline{p}(\psi)$ were greater than $VA(\mu;\psi)$, investors would on average receive less than the return required for the project's risk. This clearly can not be an equilibrium. (Investors would be better off not making an offer.) If the price $\underline{p}(\psi)$ were less than $VA(\mu;\psi)$, excess returns would exist for investors so that it would pay some investor to offer a slightly higher price less than $VA(\mu;\psi)$. In this way competition among investors would continue until $\underline{p}(\psi)$ equals $VA(\mu;\psi)$. We finally consider the case where an investor offers a price say $\underline{p}^h(\psi_1)$ greater than $VA(\mu_1;\psi_1)$. Such a price offer could be profitable to the investor if it also attracts entrepreneurs of type $\mu_2 > \mu_1$ with higher project values $VA(\mu_2)$. This will occur if entrepreneurs of type μ_2 prefer to choose ψ_1 and receive a price $\underline{p}^h(\psi_1)$ for their shares rather than choose ψ_2 greater than ψ_1 and obtain a price $\underline{p}(\psi_2)$. Such a possibility exists since risk averse entrepreneurs prefer to hold a smaller proportion of shares in their own projects (that is, they prefer to be more diversified). See for instance Leland and Pyle (1977) p. 376).

Before we examine whether such a $\underline{p}^h(\psi_1)$ is profitable note that equation (4.5) suggests that $VA(0) = \bar{K}$ and $VA(\psi) > \bar{K}$ for $\psi > 0$. Since in equilibrium a project is only undertaken if $\psi > 0$ (see Leland and Pyle, 1977, p. 379) it follows that for all projects that are undertaken, the market value given μ exceeds the cost $\bar{K}$. Consequently, there exist projects with market values less than cost that are not undertaken in equilibrium.

We next examine if there exist opportunities for potential gain by price searching investors. Riley (1979) examines precisely this issue. He shows that under our assumptions of a large number of projects whose market value is less than cost, and the strict concavity of $F(\mu)$, it is not possible for an investor to profit by making an alternative offer $\underline{p}^h(\psi)$.

Therefore the strategies described earlier constitute a Nash equilibrium.

<u>Proof of Proposition 4.3.1</u>

Before proving the proposition we state and prove several lemmas.

<u>Lemma 4.3.2</u> For an entrepreneur of type μ who undertakes the project, $\partial E[U(\tilde{W}_1;\ \mu,\sigma_E,\sigma_1,\sigma_2)]/\partial\psi \le 0$ in equilibrium.

Proof of Lemma 4.3.2

Consider an entrepreneur of type μ who undertakes the project. In equilibrium investors pay a price

$$\underline{p} = \frac{1}{(1+r)}[\mu(\hat{\mu},\psi) - \lambda] \text{ if the auditor is employed}$$

$$\frac{1}{(1+r)}[\mu(\psi) - \lambda] \text{ if the auditor is not hired}$$

Substituting into equation (4.14) and recognizing

$$\mu(\hat{\mu},\psi) = \mu \text{ if the auditor is employed}$$

$$\mu(\psi) = \mu \text{ if the auditor is not hired}$$

we obtain

$$E[U(\tilde{W}_1;\mu,\sigma_E,\sigma_1,\sigma_2)] = G[\psi\lambda+\nu[\tilde{M} - (1+r)V_M] + [\tilde{W}_0-\tilde{R}-g(c)]\ (1+r)$$

$$+ \mu - \lambda - \frac{a}{2}[\psi^2\sigma_v^2 + \nu^2\sigma_M^2 + 2\psi\nu\text{cov}(\tilde{v},\tilde{M})] \quad (4.18)$$

where $g(c) = 0$ if the auditor is not hired. Differentiating (4.18) with respect to ν, we have

$$\nu = \frac{\lambda^*}{a} - \frac{\psi\ \text{cov}(\tilde{v},\tilde{M})}{\sigma_M^2}$$

Substituting into Equation (4.18) we obtain

$$E(U(\tilde{W}_1;\mu,\sigma_E,\sigma_1,\sigma_2] = G[\frac{\lambda^*}{a}[\tilde{M} - (1+r)V_M] + [\tilde{W}_0 - \bar{K} - g(c)](1+r)$$

$$+ \mu - \lambda - \frac{a}{2}\sigma_M^2 \frac{\lambda^{*2}}{a^2} - \psi^2 \frac{a}{2} Z]$$

Therefore $\dfrac{\partial E[U(\tilde{W}_1;\mu,\sigma_E,\sigma_1,\sigma_2]}{\partial\psi} = -a\psi\bar{Z}G'(\cdot) \leq 0$ since $a,\psi,G'(\cdot) > 0$ and

$\bar{Z} = \dfrac{\sigma_v^2\sigma_M^2 - [\text{cov}(\tilde{v},\tilde{M})]^2}{\sigma_M^2} \geq 0.$ $\dfrac{\partial E[U(\tilde{W}_1;\mu,\sigma_E,\sigma_1,\sigma_2)]}{\partial\psi}$ is strictly less than zero if $\bar{Z} > 0$, which is the case when the project and market returns are not perfectly correlated. Q.E.D.

The lemma is intuitive. It states that if the entrepreneur has the choice of two levels of ψ (say ψ_1 and ψ_2) to signal μ, he prefers the smaller. The only reason the entrepreneur holds ψ is to ensure that investors do not undervalue the firm. If alternative mechanisms exist to communicate μ in a believable way to investors, the entrepreneur prefers to hold as little ψ as possible.

In the next lemma we demonstrate that, in equilibrium, the audit report may alter the investors' assessments about the support of μ. We later show that this is a necessary condition for auditing to be ex-ante incentive efficient.

Lemma 4.3.3

$$\lim_{\psi \to 0^+} \mu(\hat{\mu},\psi) = \max\,(\hat{\mu}-u(c),\ (1+r)\,\bar{K} + \lambda)$$

<u>Proof of Lemma 4.3.3</u>

Given μ, the audit report $\hat{\mu}$ will be in the interval $[\mu, \mu+u(c)]$. Therefore $\hat{\mu}-u(c) \in [\mu-u(c), \mu]$ so $\hat{\mu}-u(c) \leq \mu$. In particular $\hat{\mu}-u(c)$ is a lower bound of the investors' support for μ.

<u>Claim 1</u> If $\hat{\mu}-u(c) > (1+r)\bar{K}+\lambda$

$$\lim_{\psi \to 0^+} \mu(\hat{\mu}, \psi) = \hat{\mu}-u(c)$$

We verify that $\lim_{\psi \to 0^+} \mu(\hat{\mu}, \psi)$ can not be strictly greater than $\hat{\mu}-u(c)$.

Suppose $\lim_{\psi \to 0^+} \mu(\hat{\mu}, \psi) = \hat{\mu}-u(c)+\bar{\delta}$ where $\bar{\delta} > 0$. Then for all realizations of $\mu \in [\mu+u(c)-\bar{\delta}, \mu+u(c)]$ which occur with positive probability)

$$\lim_{\psi \to 0^+} \mu(\hat{\mu}, \psi) = \hat{\mu} - u(c) + \delta > \bar{\mu}.$$

which contradicts informational consistency of the equilibrium.

<u>Claim 2</u> If $(1+r)\bar{K} + \lambda > \hat{\mu} - u(c)$

$$\lim_{\psi \to 0^+} \mu(\hat{\mu}, \psi) = (1+r)\bar{K} + \lambda.$$

$\hat{\mu} - u(c)$ is, as in the earlier claim, a lower bound of the investors' support for μ. However, from our discussion in Section 4.2 it follows that for $\psi > 0$, $(1+r)\bar{K} + \lambda$ is also a lower bound of the investors assessment about μ. Therefore $\lim_{\psi \to 0^+} \mu(\hat{u},\psi) \geq (1+r)\bar{K} + \lambda > \hat{u} - u(c)$. As in the earlier claim, $\lim_{\psi \to 0^+} \mu(\hat{u}, \psi) > (1+r)\bar{K} + \lambda$ contradicts informational consistency of the equilibrium. Therefore $\lim_{\psi \to 0^+} \mu(\hat{u}, \psi) = (1+r)\bar{K} + \lambda$. We have therefore proven the Lemma.

$$\lim_{\psi \to 0^+} \mu(\hat{u}, \psi) = \max(\hat{u}-u(c), (1+r)\bar{K} + \lambda).$$

We are now in a position to prove Proposition 4.3.1.

<u>Proof of Proposition 4.3.1</u>

Notice if the auditor is not employed the equilibrium is exactly as described in Proposition 4.2.1. Therefore, we focus only on the price schedule offered by investors if the auditor is employed by the entrepreneur.

<u>VERIFICATION OF THE ENTREPRENEUR'S STRATEGY</u>

The entrepreneur's problem is to $\underset{c,\psi,\nu}{\text{Maximize}}\ E[U(\tilde{W}_1); \mu, \sigma^*_E, \sigma^*_1, \sigma^*_2]$ given the investors' strategies,

$$\underline{p}(\psi) = VA(\mu;\psi) = \frac{1}{(1+r)} [\mu(\psi) - \lambda] \qquad \text{without auditing,}$$

$$\underline{p}(\psi) = VA(\mu; \hat{u}, \psi) = \frac{1}{(1+r)} [\mu(\hat{u}, \psi)-\lambda] \qquad \text{with auditing.}$$

Differentiating equation (4.14) with respect to ψ and ν after substituting for p as defined above yields

$$G' \cdot [\mu - \mu(\hat{u},\psi) + \lambda + (1-\psi)\mu_{\psi} - a\psi\sigma_v^2 - a\nu cov(\tilde{v},\tilde{M})] = 0 \quad (4.20)$$

$$G' \cdot [\bar{M} - (1+r)V_M - a\nu\sigma_M^2 - a\psi cov(\tilde{v},\tilde{M})] = 0 \quad (4.21)$$

Substituting for $a\nu$ from (4.21) and using the condition $\mu(\hat{u},\psi) = \mu$ enables us to derive the following differential equation for the valuation schedule $\mu(\hat{u},\psi)$.

$$(1-\psi)\mu_{\psi} = a\psi\bar{Z} \quad (4.22)$$

where $$\bar{Z} = \frac{\sigma_v^2\sigma_M^2 - [cov(\tilde{v},\tilde{M})]^2}{\sigma_M^2} \geq 0$$

The solution to the differential equation (4.22) is a family of functions

$$\mu(\hat{u},\psi) = -a\bar{Z}[\log(1-\psi) + \psi] + C_1 \quad (4.23)$$

where C_1 is an arbitrary constant. It can be verified that schedules in the family satisfy the second-order conditions for expected utility maximization.

By lemma 4.3.3 $\lim_{\psi \to 0^+} \mu(\hat{u},\psi) = \max(\hat{u}-u(c), (1+r)\bar{K}+\lambda) = C_1$

since $[\log(1-\psi)+\psi] \to 0$

Therefore $\mu(\hat{\mu},\psi) = -a\bar{Z}[\log(1-\psi)+\psi] + \max(\hat{\mu}-u(c),(1+r)\bar{K} + \lambda)$ (4.16)

As in the previous section, given $\hat{\mu}$, $\mu(\hat{\mu},\psi)$ is a strictly increasing function of ψ. Consequently, investors can invert it to determine precisely the entrepreneur's μ and the project's value $VA(\hat{\mu},\psi)$. Investors read higher entrepreneurial ownership as a signal of a more favorable project. In turn the entrepreneur chooses a higher fraction of ownership in projects with a higher μ.

We have therefore verified that if the entrepreneur employs the auditor, the utility maximizing action for the entrepreneur given the strategies of the investors is to choose ψ (and ν) according to equation (4.16). That is, conditional on the auditor being employed choosing ψ according to (4.16) is a best response.

We finally verify that there exists a cut off level for μ (denoted by μ_o) such that for values of μ less than μ_o the entrepreneur chooses not to hire the auditor whereas for values of μ greater than μ_o, the entrepreneur employs the auditor and pays him a fee $g(c)$ to issue a report $\hat{\mu}$.

We show that $E_{\hat{\mu}|c}[E_{\tilde{\nu},\bar{M}}[U(\tilde{W}_1)|\hat{\mu},\psi_A]] - E_{\tilde{\nu},\bar{M}}[U(\tilde{W}_1)|\psi_S]$ is monotonically increasing in μ, where ψ_A and ψ_S denote holdings of the project in the equilibrium with auditing and the equilibrium without auditing respectively. Consequently for all $\mu \geq \mu_o$, employing the auditor is optimal.

$$E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\psi_s] = G\{\psi_s[\mu-\mu(\psi_s)+\lambda] + \nu_s[\bar{M}-(1+r)V_M] + (1+r)(\bar{W}_o-\bar{k})$$

$$+ \mu(\psi_s) - \lambda - \frac{a}{2}\psi_s^2\sigma_v^2 - \frac{a}{2}\nu_s^2\sigma_M^2 - a\psi_s\nu_s \text{cov}(\tilde{v},\tilde{M})\}$$

where $\mu(\psi_s) = -a\bar{Z}[\log(1-\psi_s) + \psi_s] + (1+r)\bar{K} + \lambda$

$$\frac{d}{d\mu} E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\psi_s] = \frac{\partial E[U(\tilde{W}_1)|\psi_s]}{\partial \mu} + \frac{\partial E(U(\tilde{W}_1)|\psi_s]}{d\psi_s}\frac{d\psi_s}{d\mu}$$

$= G'(\cdot)\psi_s$ (since $\frac{\partial E[U(\tilde{W}_1)|\psi_s)}{d\psi_s} = 0$ by the First Order Condition).

$$E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\hat{\mu},\psi_A] = G\{\psi_A[\mu-\mu(\hat{\mu},\psi_A) + \lambda] + \nu_A[\bar{M}-(1+r)V_M]$$

$$+ (1+r)(\bar{W}_o-\bar{K}-g(c)) + \mu(\hat{\mu},\psi) - \lambda$$

$$- \frac{a}{2}\psi_A^2\sigma_v^2 - \frac{a}{2}\nu_A^2\sigma_M^2 - \psi_A\nu_A \text{cov}(\tilde{v},\tilde{M})\}$$

where $\mu(\hat{\mu},\psi_A) = -a\bar{Z}[\log(1-\psi_A) + \psi_A] + \max(\hat{\mu} - u(c), (1+r)\bar{K} + \lambda)$

We next consider $\frac{\partial}{\partial\mu} E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\hat{\mu},\psi_A]$

Case 1 $\hat{\mu} - u(c) < (1+r)\bar{K} + \lambda$

In this case $\psi_A = \psi_s$ and

$$\frac{\partial}{\partial\mu} E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\hat{\mu},\psi_A] = G'(\cdot)\psi_s \quad \text{(as above)}$$

<u>Case 2</u> $\hat{\mu} - u(c) \geq (1+r)\bar{K} + \lambda$

where $\hat{\mu} = \mu + \bar{r}u(c)$, $\bar{r} \in [0,1]$ since $\hat{\mu} \in [\mu, \mu+u(c)]$

$$\frac{\partial}{\partial\mu} E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\hat{\mu},\psi_A] = G'(\cdot)(1-\psi_A)\frac{\partial(\hat{\mu}-u(c))}{d\mu} + G'(\cdot)\psi_A \frac{d\mu}{d\mu}$$

$$+ \frac{\partial E[U(\tilde{W}_1)|\hat{\mu},\psi_A]}{\partial\psi_A} \frac{d\psi_A}{d\mu}$$

$$= G'(\cdot)$$

(since by the First order condition

$$\frac{\partial E\ [U(\tilde{W}_1)|\hat{\mu},\psi_A]}{d\psi_A} = 0)$$

Thus, for every possible realization of $\hat{\mu} - u(c)$ (or alternatively $\bar{r}$)

$$\frac{\partial}{\partial\mu} \{E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\hat{\mu},\psi_A] - E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\psi_s]\} \geq 0.$$

Indeed since $\hat{\mu} - u(c) > (1+r)\bar{K} + \lambda$ for some $\hat{\mu}$ (otherwise employing an auditor would never be optimal),

$$\frac{\partial}{\partial\mu} \{E_{\tilde{\mu}|c}[E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\hat{\mu},\psi_A]] - E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)|\psi_s]\} > 0$$

For values of μ such that $\mu < (1+r)(\bar{K} + g(c)) + \lambda$, the entrepreneur obviously does not find it worthwhile to employ the auditor. As the benefit to the entrepreneur from employing the auditor increases monotonically with μ, there exists a μ (denoted by μ_o) at which the entrepreneur is indifferent between employing and not employing the auditor. In other words, for values of $\mu < \mu_o$ the entrepreneur's expected utility from not hiring the auditor exceeds his expected utility from employing the auditor, whereas for all values of $\mu > \mu_o$ the entrepreneur's expected utility is larger when the auditor is employed. That is, for $\mu < \mu_o$, the entrepreneur's optimal strategy is not to hire the auditor whereas for $\mu \geq \mu_o$ employing the auditor is optimal.

The intuition behind this result is as follows. In the equilibrium without auditing, the entrepreneur holds increasing proportions of shares in his own project in order to signal increasing values of μ. The higher risk that the entrepreneur bears to signal the higher μ decreases the entrepreneur's expected utility at an increasing rate relative to the first best case. This provides increasing returns to auditing as a function of μ.

We next verify that each investor's strategy is a best response.

VERIFICATION THAT EACH INVESTOR'S STRATEGY IS A BEST RESPONSE

If the entrepreneur does not employ the auditor each investor's strategy is exactly as described in Proposition 4.2.1. Therefore, we focus on each investor's strategy when the entrepreneur employs the auditor. As in the previous proposition, each investor j can, given $\hat{\mu}$ and ψ and the entrepreneur's strategy, determine the entrepreneur's value of μ and the project value $VA(\mu;\hat{\mu},\psi)$ using equation (4.16). Using arguments similar to those used in the proof of Proposition 4.2.1 it follows that each investor j's best response strategy is to offer a price schedule $\underline{p}_j(\hat{\mu},\psi) = VA(\mu;\hat{\mu},\psi) = \frac{1}{(1+r)}[\mu(\hat{\mu},\psi) - \lambda]$ as in equation (4.17B). We therefore conclude that the strategies constitute a Nash equilibrium.

Q.E.D.

<u>Proof of Proposition 4.4.1</u>

We do not describe the complete set of equilibrium strategies in the game. Indeed the price schedules $\underline{p}_j(\cdot)$ offered by investors will now depend on $\hat{\mu}$, ψ and c, the entrepreneur's choice of auditor. Instead, we verify that in an informationally consistent equilibrium the entrepreneur's expected utility is greater if $\bar{c}$ is chosen rather than c.

$$\underset{\tilde{v},\tilde{M}}{E}[U(\tilde{W}_1)|c,\hat{\mu},\psi] = G(\psi[\mu - \mu(c,\hat{\mu},\psi)+\lambda] + \nu[\bar{M}-(1+r)v_M]$$

$$+ (1+r)(\bar{W}_o-\bar{K}-g(c)) + \mu(c,\hat{\mu},\psi) - \lambda - \frac{a}{2}\psi^2\sigma_v^2 - \frac{a}{2}\nu^2\sigma_M^2 - \psi\nu\text{cov}(\tilde{v},\tilde{M}))$$

$$\text{where } \mu(c,\hat{\mu},\psi) = -a\bar{Z}[\log(1-\psi)+\psi] + \max(\hat{\mu}-u(c),\ (1+r)\bar{K}+\lambda) \qquad (4.16)$$

Differentiating w.r.t. $\hat{\mu} - u(c)$ we have

$$G_{\hat{\mu}-u(c)}(\cdot) = (1-\psi)G'(\cdot) > 0 \qquad \text{for } \hat{\mu}-u(c) \geq \bar{K}(1+r)+\lambda$$

$$= 0 \qquad \text{for } \hat{\mu}-u(c) < \bar{K}(1+r)+\lambda$$

We next compare the entrepreneur's expected utility from choosing an auditor of quality $\bar{c}$ rather than c.

$$E_{\tilde{\mu}|\bar{c}}[E_{\tilde{v},\bar{M}}[U(\tilde{W}_1)]] - E_{\tilde{\mu}|c}[E_{\tilde{v},\bar{m}}[U(\tilde{W}_1)]]$$

$$= \int_{\mu-u(c)}^{\mu} G(\cdot)[\bar{f}(\hat{\tau}) - f_1(\hat{\tau})]d\hat{\tau}$$

where $\hat{\tau} = \hat{\mu} - u(c)$

$$= G(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]\Big|_{\mu-u(c)}^{\mu} - \int_{\mu-u(c)}^{\mu} G_{\hat{\tau}}(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]d\hat{\tau}$$

$$= -\int_{\mu-u(c)}^{\mu} G_{\hat{\tau}}(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]d\hat{\tau}$$

We consider two cases. The first case is when $\hat{\mu}-u(c)$ is greater than or equal to $(1+r)\bar{K} + \lambda$ for all realizations of $\hat{\mu}$.[10] The second case is when $\hat{\mu} - u(c)$ is less than $(1+r)\bar{K} + \lambda$ for some realization of $\hat{\mu}$. In each case we examine the sign of $-\int_{\mu-u(c)}^{\mu} G_{\hat{\tau}}(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]d\hat{\tau}$.

<u>Case 1</u>: $\hat{\mu} - u(c) \geq (1+r)\bar{K} + \lambda$ for all $\hat{\mu}$.

In this case it immediately follows that

$$- \int_{\mu-u(c)}^{\mu} G_{\hat{\tau}}(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]d\hat{\tau} \geq 0$$

since $G_{\hat{\tau}}(\cdot) > 0$ and $[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})] \leq 0$.

Since the two density functions $\bar{f}$ and f_1 are not identical $\bar{f}$ will be strictly preferred to f_1, that is $E_{\tilde{\mu}|\bar{c}}[E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)]] > E_{\tilde{\mu}|c}[E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)]]$ in equilibrium.

<u>Case 2</u> $\hat{\mu} - u(c) < (1+r)\bar{K} + \lambda$ for some $\hat{\mu}$.

We rewrite

$$- \int_{\mu-u(c)}^{\mu} G_{\hat{\tau}}(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]d\hat{\tau}$$

$$= - \int_{\mu-u(c)}^{\bar{K}(1+r)+\lambda} G_{\hat{\tau}}(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]d\hat{\tau}$$

$$- \int_{\bar{K}(1+r)+\lambda}^{\mu} G_{\hat{\tau}}(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]d\hat{\tau}$$

$$= - \int_{\bar{K}(1+r)+\lambda}^{\mu} G_{\hat{\tau}}(\cdot)[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})]d\hat{\tau}$$

(since $G_{\hat{\tau}}(\cdot) = 0$ for $\hat{\tau} = \hat{\mu}-u(c) < \bar{K}(1+r)+\lambda$)

≥ 0 since $G_{\hat{\tau}}(\cdot) > 0$ and

$$[\bar{F}(\hat{\tau}) - F_1(\hat{\tau})] \leq 0 \quad \text{for} \quad \hat{\tau} = \hat{\mu} - u(c) \geq \bar{K}(1+r)+\lambda$$

Since the two density functions $\bar{f}$ and f_1 are not identical over the range $(\bar{K}(1+r)+\lambda, \mu)$, $\bar{f}$ will be strictly preferred to f_1, that is $E_{\tilde{\mu}|\bar{c}}[E_{\tilde{v},\tilde{M}}[U(\tilde{W}_1)] > E_{\tilde{u}|c}[E_{\bar{v},\tilde{M}}[U(\tilde{W}_1)]]$ in equilibrium. We therefore conclude that the entrepreneur prefers to employ an auditor of quality $\bar{c}$ rather than c.

Q.E.D.

ENDNOTES

1 This is a natural assumption since shares in a project can only be sold after the entrepreneur initiates the project.

2 We restrict the outcome $\mu + \tilde{v}$ to be shared linearly by the entrepreneur and investors. We assume that financing arrangements are geared to equity shares and that the transactions costs of writing a contract and verifying $\mu + \tilde{v}$ are very large.

3 We could have written investor j's utility as some monotonic concave function of the payoffs. However maximizing utility is equivalent to maximizing investor j's payoff as defined above.

4 The concept of second-best here is analogous to the second-best sharing rule described in Chapter 3. There the optimal second best contract is inefficient relative to the first-best case where the manager's action can be costlessly observed.

5 As in Chapter 3, by the auditor working hard and choosing the action c we mean both (i) the auditor supplying a high level of effort (ii)truthfully reporting what he finds.

6 We assume that since the entrepreneur knows μ perfectly, he can always convince the auditor that μ has occurred. Therefore, the audit report never reports a value less than μ. This assumption is not critical to the analysis but the fact that $\tilde{\hat{\mu}}$ has moving support is a necessary condition for the audit to improve efficiency.

7 In a multi-period model we would have also explicitly modeled the auditor's strategic decision to work/shirk and the investors' strategic decision to investigate. Since we consider only a single period model here we assume that reputation and penalty considerations ensure, as in Chapter 3, that the auditor works hard and provides a level of audit quality c in this period.

8 Feltham and Hughes (1985) obtain a similar result. In their model, the Nash equilibrium contract is independent of the investors' prior beliefs.

9 Our discussion above assumes linear shares. Our intuition is that even had we considered optimal sharing arrangements, a necessary condition for auditing to be useful would be that the audit alters the support of the investors' probability distribution about μ. We conjecture that what drives the result is the requirement of informational consistency rather than the linear sharing rule.

10 We define
$$\hat{F}(\hat{\mu} - u(c)) = 0 \text{ for } \hat{\mu} - u(c) \in [\mu - u(c),\ \mu - u(\bar{c})]$$
$$> 0 \text{ for } \hat{\mu} - u(c) \in [\mu - u(\bar{c}),\ \mu]$$
$$= 1 \text{ for } \hat{\mu} - u(c) = \mu.$$

[11] In fact, the assumption can be weakened so that the probability function f is at least as large as f_1 in the sence of second-order stochastic dominance.

[12] The condition that the audit cost is the same for c and $\bar{c}$ can also be relaxed, as we discuss later.

CHAPTER 5

SUMMARY AND CONCLUSION

We have presented two models of auditing. The first is a multi-period model of auditing with incomplete information. An owner contracts with a manager to take actions on the owner's behalf. The outcomes of the manager's actions are unobservable to the owner. The auditor's role is to investigate and provide information to the owner about the actual outcomes. The auditor is modeled as a rational economic agent taking investigative acts and making reports under moral hazard. This allows us to address issues of reputation formation and auditor independence that reduce the auditor's moral hazard problem. The second model is one of adverse selection. The auditor's role is to reduce information asymmetry among the informed entrepreneur and uninformed investors. We provide necessary and sufficient conditions for auditing to be useful.

In the first model, we examine optimal strategies for the owner, manager and auditor. The auditor's preference is to shirk. However, reputation effects ensure that the auditor's optimal strategy over a large number of periods is to work hard and report truthfully. We also show that creating incentives to build a reputation reduces the investigation costs the owner must incur to monitor the auditor. Reputation formation serves as a partial substitute for costly monitoring by the owner.

We examine various definitions of auditor independence proposed by Antle (1980). Independence (1) is a statement about how the auditor's report affects his payments. Independence (1) is satisfied in our model in that the audit payment is independent of the auditor's report. Independence (2) requires the auditor to work hard and report truthfully. This is satisfied in equilibrium over all but a few periods in the game. Independence (3) states that the auditor manager subgame is played non-cooperatively. We show that multi-period reputation effects preclude cooperative and collusive play in the auditor-manager subgame over many periods of the game.

We employ the notion of sequential equilibrium throughout our analysis. Since such equilibria are difficult to compute and characterize, we restrict ourselves to binary outcomes and binary action choices for the auditor and manager. Extending the analysis to more general settings remains.

A second area for further study is the examination of the manager's incentives to build reputations. Such models are extremely complicated and techniques for solving such models are not well developed.[1]

Another interesting extension is a general equilibrium analysis of auditing with different auditor types. Such a model could be used to address issues of reputation and competition among auditors. We conjecture that in an equilibrium with reputations, the partial pooling of 'weak' and 'strong' auditors may dissipate rents to the strong auditor and consequently reduce the supply of strong auditors in the market place. This may, in turn, alter the incentives for weak auditors to build reputations.

In the second model, we obtain a necessary and sufficient condition for auditing to be useful to the entrepreneur. The condition is that the audit report alter the support of the investors' probability distribution about μ. For expositional clarity we consider only a single period of the multiple periods in which the auditor is hired. Consequently, we do not model the effect on auditor behavior of reputation and penalty considerations as we do in the first model.

We introduce a second auditor to examine the entrepreneur's strategic choice of auditor. In general, the choice of auditor depends on the audit cost and the productive effect of audit quality. We show conditions under which heterogeneous entrepreneurs choose the same audit quality.

Of course, several simplifying assumptions are made. We assume that the sharing rule among the entrepreneur and investors is linear. An interesting extension is to examine optimal sharing arrangements among the entrepreneur and investors and the role of auditing within such a context. A second area for further study is the effect on the signaling equilibrium of reputation formation by the entrepreneur.

ENDNOTES

1 See Kreps and Wilson (1982a).

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